Aggregate and Distributional Implications of a Military Buildup*

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Abstract

We study the aggregate and distributional implications of a permanent increase in government spending of the magnitude implied by the 2025 change in NATO "core defence" spending target. Our framework of analysis is a calibrated Overlapping-Generations model with Heterogeneous Agents and a rich fiscal side that includes fully specified tax-and-transfer and pay-as-you-go social-security systems. We examine how alternative fiscal adjustments to the shock shape macroeconomic outcomes, aggregating them up from the distributions of individual labour-supply and consumption responses. We highlight the presence of a tradeoff, when choosing among fiscal adjustments, between mitigating *aggregate* crowding-out of private consumption versus reducing consumption *inequality*.

Keywords: Overlapping generations; Heterogeneous agents; Government spending; Inequality.
JEL Codes: C68, D15, J11.

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1 Introduction

Following the growing military threat from Russia and the uncertainties surrounding the U.S. defence umbrella, European leaders have recently pledged to significantly ramp up their military capabilities. This process culminated in NATO members' commitment in June 2025 to raise their spending on "core defence requirements" to 3.5% of GDP, up from the previous 2% target.[1] This commitment occurs within the context of already rising military spending over the last decade (see Figure 1) and, for a number of countries, against the backdrop of limited fiscal space. This paper assesses the distributional and aggregate implications of a shock to government spending of this magnitude and persistence. In particular, we examine how its financial burden can be distributed across the population, how it determines individual incentives to work, consume, and save across households, and how these choices ultimately shape macroeconomic outcomes.

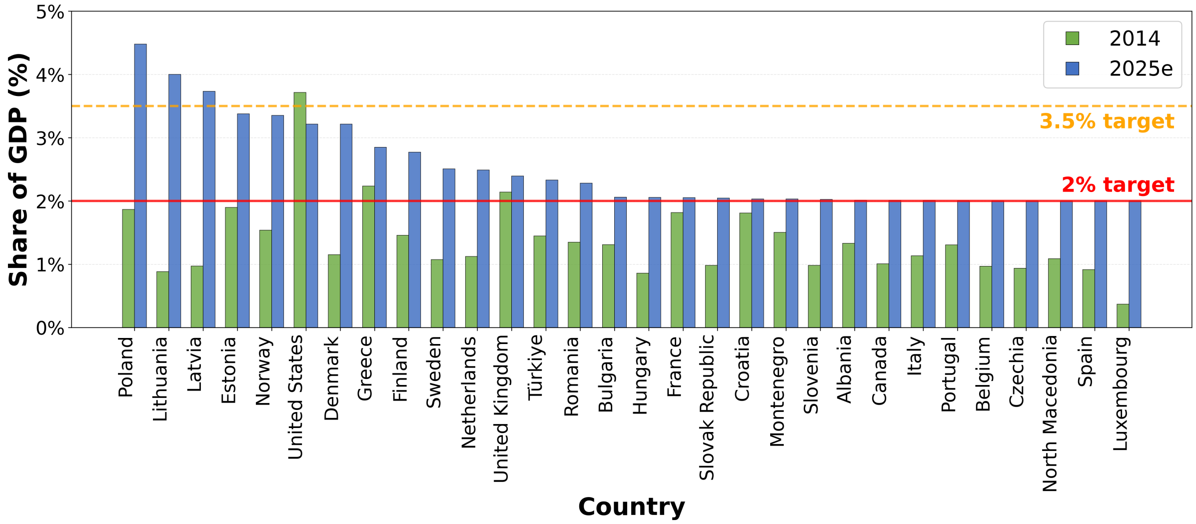
Our tool of investigation is a rich quantitative Overlapping-Generations model with Heterogeneous Agents ("OLG-HA") in which households' individual state and decisions differ both across and within cohorts. Households differ across cohorts due to the age profile of individual productivity and life-cycle behaviour, and within cohorts due to idiosyncratic productivity (both transitory and permanent) at any given age. Those features, together with the tax and social-security systems and the returns on assets, determine the payoffs associated with all labour-supply, consumption, and portfolio decisions. Accordingly, the model incorporates a rich fiscal side that includes, besides government spending and public debt, various taxes (on consumption, labour, and capital), social transfers, and a pay-as-you-go pension system. We model the government spending shock as permanent (consistent with the spirit of the NATO commitment) and focus on its medium- to long-run impact.[2] This sets our paper apart from existing studies (reviewed below) of the macroeconomic impact of government spending shocks, which either abstract from distributional considerations altogether or focus on the *short-run* government spending multiplier. We calibrate the model to the French economy, but rely primarily on international datasets for broad applicability.

We endow our model with three features that we believe are key to addressing our question. First, we model labour-supply decisions along the *extensive* margin so that each individual self-selects into the labour force, as in Chang and Kim (2007). Incentives to join the labour force vary widely across households and are shaped by the tax-and-transfer system; this makes the macroeconomic impact of the government spending shock contingent on how its burden is distributed among taxpayers. Given the importance of individual labour-market participation choices for fiscal transmission, we calibrate the model to match key moments of their distribution. For instance, we calibrate labour-market participation rates, individual hours worked and hourly earnings by age and skill groups to match their empirical counterparts (from suitable Euro Area surveys), and we recover idiosyncratic

[1] See The Hague Summit Declaration by NATO Heads of State and Government participating in the meeting of the North Atlantic Council on 25 June 2025. The new NATO guidelines also involve an additional 1.5% of GDP on "defence- and security-related spending", but for which the distinction with general government spending is more fuzzy.

[2] According to Marzian and Trebesch (2026), peacetime military buildups typically have a permanent component, unlike wartime buildups.

Figure 1: NATO guidelines versus actual spending on core defence requirements.



labour-earnings risk by imputation from GRID (Guvenen et al., 2022a). We also verify that our calibration produces reasonable marginal propensities to earn (*MPE*s) and labour-participation elasticities (*LPE*s) by age and skill groups, so that we adequately capture how wealth and substitution effects on labour supply play out across the household distribution.

The second feature we introduce is a warm-glow bequest motive, which urges households to accumulate assets beyond what is needed for consumption smoothing, following De Nardi (2004). As explained by Straub (2019), the bequest motive explains why high-skilled individuals accumulate wealth faster than lower-skilled ones (the former not only have higher permanent income but also higher saving rates), eventually generating high wealth concentration at the top. Wealth dispersion critically matters for the transmission of fiscal shocks for at least two reasons. For one, the wealth distribution directly maps into the distribution of wealth effects on labour supply and associated MPEs, because richer individuals are relatively less sensitive to a given income loss than poor individuals. Next, it matters more specifically for the incidence of capital taxation on the distribution of disposable income, since richer individuals are relatively more exposed to this tax. To capture such effects in a quantitatively meaningful manner, we make sure to calibrate the strength of the bequest motive to match the observed distribution of (bequeathed) wealth in the population.

Third, we assume that households not only have an unconditional taste for wealth in the form of the bequest motive, but also a specific preference for government bonds (i.e., "liquid wealth") over capital claims. In the data, the annual return to capital is high (about 8% pre-tax in the Euro Area, according to Marx et al., 2021), while the annual real return on government bonds is comparatively low (somewhere between 1% and 2% at the time of writing). Such a return gap would be difficult to rationalise on the basis of aggregate risk alone (which is absent from our analysis anyway). Yet, not accounting for this gap would either lead us to underestimate the return to capital in asset portfolios or overestimate the cost of public debt, thereby misrepresenting the government's true fiscal capacity. Accordingly, we follow Krishnamurthy and Vissing-Jorgensen (2012) in relying on a "convenience yield" on liquid assets to produce the required return differential.[3]

With those considerations in mind, we examine several potential fiscal adjustments following the military buildup. The government can adjust the parameters of the social security system by increasing the legal retirement age or reducing pensions. Another possibility is to raise revenue via taxation, whether on labour, capital, or consumption. We consider these policies both in isolation and in conjunction, holding the path of government spending-to-GDP identical across specifications. For conciseness, we focus our analysis on two classes of financing scenarios, depending on whether *one* or *two* fiscal instruments are adjusted. The first class (involving one instrument) provides clear insights into how each instrument affects the equilibrium. This allows the macroeconomic effects of each instrument to be decomposed into the underlying distributions of labour supply and consumption across the population. It also permits the construction of unambiguous Laffer curves for each tax, thereby revealing their potential self-destructiveness. The second class of scenarios, involving two instruments, yield arguably more plausible fiscal adjustments, and our model provides a means to evaluate their effectiveness by various criteria. One such criterion is how much total private consumption is "crowded out" by public spending — a typical summary measure of the cost of public-spending expansions in Representative-Agent economies. Another criterion is the impact of alternative adjustments on consumption *inequality*, beyond aggregate crowding-out.

Our main results are as follows. Starting with fiscal adjustments based on a single instrument, we note that increasing the legal retirement age by two years (in our calibration, from age 63, the current legal age in France, to age 65), while keeping all other fiscal instruments unchanged, allows the government to raise spending by 1.37 pp. of GDP (close to the new NATO guidelines and in the ballpark of policy discussions) without any consumption crowding out, whether aggregate or individual. In this scenario, higher government spending is financed by greater labour-market participation, primarily (but not only) by those directly affected by the shift in the legal retirement age. For comparability across financing scenarios, we use the path of additional government spending-to-GDP allowed by this simple pension reform as a benchmark, imposing it in all alternative fiscal adjustments.

Next, we consider a financing scenario in which the exact same path of government spending to GDP is entirely financed via labour-income taxation, by adjusting the level or progressivity of a tax schedule a la Benabou (2000) and Heathcote et al. (2017). Both forms of labour-income taxation come with their own distortions. By discouraging the labour supply of low-productivity workers, an increase in broad-based taxation destroys the tax base and generates considerable aggregate consumption crowding out, while also substantially raising consumption inequality. Increasing tax *progressivity* instead effectively avoid such dropouts (thanks to the low LPEs of high-income households) but (i) depletes aggregate savings (which are dominated by high-income households) and ultimately the capital stock and (ii) still pushes out out of the labour force few but highly productive

[3]See also Fisher (2015), Del Negro et al. (2017), Anzoategui et al. (2019) and Challe and Matvieiev (2024) for alternative formulations of this assumption. One contribution of the present paper is to articulate the bequest and liquidity motives within the flow-utility functional in a way consistent with households holding symmetric portfolios in equilibrium, despite heterogeneous asset wealth. This avoids expanding the individual state space in a way that would otherwise complicate the numerical resolution of the model.

workers, thereby harming Total Factor Productivity ("TFP" henceforth). Eventually, the two labour tax parameters prove similarly damaging to total private consumption, though they differ sharply in who bears the brunt; namely, low-consumption households after a hike in broad-based taxation, high-consumption households if progressivity is used instead.

We run similar computations for the other single-instrument policies, such as cuts in retirement benefits or hikes in capital or consumption taxation.[4] In either case, we similarly aggregate up the responses of employment, capital, TFP, output and consumption from changes in individual participation and portfolio choices, trace those changes back to the underlying distortions in the tax-and-transfer system, and ultimately highlight the implied trade-off between aggregate consumption crowding out and consumption inequality. We notably find that capital taxation is the second-worst form of taxation (after labour taxation) in terms of aggregate consumption crowding out, due to its distortionary impact on capital accumulation. Moreover, it yields only moderate gains in terms of consumption inequality, making it a poor contender overall for financing the buildup. The other two instruments, and especially the generosity of retirement benefits, strike a much better balance between minimising aggregate consumption losses versus their distribution across the population.

In the final step of our analysis, we examine a set of mixed fiscal adjustments using two instruments. For conciseness, we focus on adjustments involving a one-year shift in the legal retirement age, combined with another instrument to cover the fiscal shortfall (any other class of mixed adjustment can, of course, be analysed within our framework). In terms of macroeconomic effects, such mixed fiscal adjustments can be understood as (near-)linear combinations of the single-instrument adjustments just discussed. In particular, they retain the benefits of shifting the legal retirement age (though in a muted manner), but also inherit the distortions that accompany the complementary tax instrument. Aggregate and distributional outcomes vary significantly across fiscal-policy mixes, and we evaluate their performance using various metrics, paying particular attention to the trade-off between aggregate consumption and consumption inequality to structure our results. By that criterion, fiscal adjustments involving the legal retirement age, the generosity of retirement benefits, or the progressivity of labour taxation clearly stand out, though they are located at very different points of the "aggregate versus inequality" tradeoff.

1.1 Literature review

The paper most related to ours is by Ferriere and Navarro (2025), who use a Heterogeneous-Agent New Keynesian (HANK) model to show that financing a short-run spending shock via progressive taxation yields larger multipliers than broad-based taxation, by targeting households with relatively low LPEs. We are instead interested in the *long-run* effect of a *permanent* increase in government spending and accordingly abstract from nominal rigidities. While we agree on the beneficial impact of progressive taxation on labour supply, we also warn against its potential effects on capital accumulation and pos-

[4] According to Marzian and Trebesch (2026), historical military buildups ("guns"), including peacetime buildups, have not been associated with cuts in social transfers ("butter"). However, most buildups in their sample predate modern pay-as-you-go pension schemes, whose parameters can, in principle, be used as fiscal tools, just like tax hikes.

sibly TFP. Furthermore, we consider a much broader set of fiscal tools that includes social security parameters and non-labour taxes. Luetticke et al. (2025) similarly use a HANK framework to rationalise the impact of wars on inequality, working notably via capital destruction and inflation. We instead focus on the impact of the ongoing *peacetime* buildup (so that the latter two factors play no role) and study how different forms of financing affect household inequality in the long run.

Next, our paper revisits the traditional neoclassical analysis of large government spending shocks, typically associated with military buildups, within the Representative-Agent framework. This includes Braun and McGrattan (1993), who examine the macroeconomic effects of World Wars I and II on the United States and Great Britain; Ohanian (1997), who shows that different tax policies (on capital versus labour) after World War II and the Korean War in the U.S impacted the economy differently; Ramey and Shapiro (1998), who examine how sector-specific changes in government spending affect the economy under costly capital reallocation; Burnside et al. (2004), who feed a neoclassical model with the historical stochastic processes for government spending and taxes associated with military buildups; and McGrattan and Ohanian (2010), who evaluate the ability of a rich neoclassical model to explain the dynamics of the US economy during World War II.[5] By construction, these papers abstract from distributional considerations (and notably the distributional impact of alternative financing schemes), which we argue, in agreement with Ferriere and Navarro (2025), are crucial for understanding the propagation of government spending shocks.

Lastly, our paper relates to the analysis of social security and tax reforms when agents are heterogeneous, whether due to overlapping generations, uninsured idiosyncratic shocks, or both. Auerbach and Kotlikoff (1987)'s OLG model laid the groundwork for subsequent work introducing within-cohort heterogeneity due to idiosyncratic risk, such as Imrohoroğlu et al. (1995, 1999), who study the optimal social security replacement rate in unfunded social security systems, Huggett and Ventura (1999), who examine the distributional impact of replacing the U.S. social security system with a two-tier structure, or Conesa and Krueger (1999), who study the political feasibility of switching from pay-as-you go to fully-funded social security.[6] This class of models has also been used to study the aggregate and distributional effects of *tax* reforms (as opposed to social security reforms). For example, Nishiyama and Smetters (2005) study the effects of replacing a progressive income tax with a flat consumption tax. Ventura (1999) examines the implications of replacing the progressive taxes on labour and capital income in the U.S. with flat taxes. More recently, Cahn et al. (2026) study the aggregate and welfare effects of a fiscal rebalancing away from payroll taxes and towards consumption taxes, while Ábraham et al. (2025) study the optimality of a joint tax-and-transfer and social-security reform. This literature focuses on how to strike the best balance among alternative tax distortions at a given level of government spending, while we are instead interested in how to best share the large aggregate income loss generated by the military buildup. Accordingly, the distribution of wealth effects on labour supply in our analysis is equally as important as the substitution effects driven by tax distortions.

[5]Within a similar conceptual framework, see also Aiyagari et al. (1992); Baxter and King (1993); Uhlig (2010).

[6]Papers studying social security reforms using alternative versions of the OLG-HA models include Nishiyama and Smetters (2007); Fehr and Habermann (2008); Imrohoroğlu and Kitao (2012); Fehr et al. (2013); McGrattan and Prescott (2017).

2 Environment

Time is discrete, and we interpret the period as corresponding to one year. There are overlapping generations of households that are subject to uninsured idiosyncratic shocks to mortality, productivity and taste for leisure. Households supply labour, consume, and accumulate assets in the form of capital claims and government bonds. A representative firm produces an all-purpose good by combining capital and labour through a neoclassical production function. The government consumes, redistributes income through age- and state-contingent transfers, collects various taxes, and issues public debt. Agents have perfect foresight about aggregates. All markets are competitive, and all prices are fully flexible. We provide further details about the environment in Appendix A. While our baseline analysis uses a closed-economy model, Appendix E exhibits an open-economy variant in which equilibrium interest rates depend on a residual asset demand from the rest of the world (our results are qualitatively unchanged under plausible elasticities of interest rates to net foreign assets).

2.1 Demographics

In each period, a new cohort enters the economy; we use the index $j \in \boldsymbol{J} = \{18, 19, ..., J = 100\}$ to denote an individual's age. In what follows, we consider a demographic steady state in which survival probabilities across ages and the population shares of various ages (the "age pyramid") are constant over time, despite population growth. To be more specific, let $L_{j,t}$ the number of age-j individuals at time t, $L_t = \sum_j L_{j,t}$ total population at time t, ψ_{j+1} the probability of surviving form age j to age $j+1$, and $\mu_j = L_{j,t}/L_t$ the population share of age j. Population grows at a rate n, which is captured by an increasing inflow of newcomers over time, i.e., $L_{18,t+1} = (1+n)L_{18,t}$.

For simplicity, we assume that (i) our economy is in a demographic steady state and (ii) the latter is insensitive to fiscal policy (both assumptions are easily relaxed). Such a steady state requires consistency between survival rates and population shares, in such a way that $\mu_{j+1}/\mu_j = \psi_j/(1+n)$ for all $j \in \boldsymbol{J}$. In the data, the latter equations do not hold strictly, since there are (small) departures from the demographic steady state; following Auclert et al. (2025), we accommodate such residuals in a model-consistent way by allowing for a flow of age-j migrants $M_{j,t}$ into (or out of) the (j,t) cohort, where $M_{j,t}$ is the solution to:

$$\frac{\mu_{j+1}}{\mu_j} = \frac{\psi_{j+1}}{1+n}\left(1 + \frac{M_{j,t}}{L_{j,t}}\right). \tag{1}$$

2.2 Households

Individual productivity. Households differ in terms of individual labour productivity, which is the product of three components, each normalised so that their unconditional mean is equal to 1. First, a household is assigned a permanent productivity level (or "skills") $\omega \in \boldsymbol{\Omega}$ at birth, where ω is drawn from a stationary distribution $\pi^{\Omega}(\omega)$; when implementing the model we will allow for three skill levels, i.e. $\boldsymbol{\Omega} = \{\omega_1, \omega_2, \omega_3\}$, calibrated to match the observed dispersion in permanent income among households of different educational levels. Next, individual productivity has an age-specific component e_j;

this will be used to match the age profile of hourly labour earnings in the data. Third, households face transitory idiosyncratic productivity shocks. These shocks follows a discrete Markov chain with support $\boldsymbol{Z} = \{z_1, \ldots, z_{N_z}\}$, transition matrix Π^Z and invariant distribution $\pi^Z(z)$. This will be used to capture the residual earnings risk that households face *within* skill and age groups. Eventually, individual productivity in a period is the product of the three components, i.e. $\omega e_j z$.

Occupational status. Let $o \in \boldsymbol{O} = \{I, E\}$ define the occupational status of an individual in a particular period, where "I" stands for "inactive" and "E" for "employed". In every period, the decision to participate in the labour market balances benefits and costs. Inactive individuals receive a transfer from the government, namely, the retirement benefit if they have attained the legal retirement age, or a basic universal income transfer otherwise. Activity comes with the prospect of labour earnings, but it also entails utility costs, which take two forms. First, an individual of age j and skill group ω incurs the disutility $\kappa_{\omega,j}$ from working. The j-index is here to accommodate the hump-shaped age pattern of labour-market participation within each skill group, while the ω-index reflects the dependence of this age pattern on the skill group; both sources of heterogeneity in labour-market participation are important features of the data and will be accounted for in the calibration. Second, the decision to be active is affected by an idiosyncratic preference shock with a continuous (extreme-value) distribution. This implies that, among all households sharing the same individual state, only those whose reservation utility is below a threshold will choose to be active. Moreover, since households know the distribution of the preference shock, they can estimate their own probability of activity in the next period and make current consumption and portfolio decisions accordingly.

There is a legal retirement age, $j = J^R$, that entitles households to receive retirement benefits, but it is not mandatory, and households freely choose when to retire. However, once the legal retirement age is reached, the decision to be inactive is irreversible.

Flow utility. Let $u(\cdot)$ denote households' flow utility from consuming, holding assets, and working:

$$u(c, \omega, j, b', k', o) = \frac{\left(c + \frac{\alpha b'}{1+\tau_t^C}\right)^{1-\sigma}}{1-\sigma} + (1-\psi_{j+1})\nu_a \Gamma_t^{\varphi-\sigma} \frac{\left(\frac{k'+(1-\alpha)b'}{1+\tau_t^C}\right)^{1-\varphi}}{1-\varphi} - \mathbb{1}_{o=E} \Gamma_t^{1-\sigma} \kappa_{\omega,j}, \tag{2}$$

where $\sigma, \varphi, \nu_a \geq 0$ and $0 \leq \alpha < 1$ are constant parameters, Γ_t an exogenous productivity trend, τ_t^C the consumption tax rate and c, b', k' the household's choice variables.

The flow utility function depends not only on consumption $c \geq 0$ (with relative risk aversion coefficient σ) and labour supply (the last term in (2)) but also on the asset portfolio (b', k'), where the primed variables denote end-of-period holdings of government bonds and capital claims, respectively. The presence of assets in the utility function reflects both a *bequest* motive (parameterised by ν_a) and a *liquidity* motive (parameterised by α). To disentangle how these two motives play out, abstract from the latter temporarily by setting $\alpha = 0$. In this situation, consumption utility is $c^{1-\sigma}/(1-\sigma)$ and the second term in the RHS of (2) reflects a warm-glow bequest motive, where the utility from bequeathing is multiplied by its probability of occurrence $1-\psi_{j+1}$. The total amount bequeathed is then $a' = k' + b'$ (i.e., total end-of-period assets), which we rescale it by $1/(1+\tau_t^C)$ to reflect the valuation of the quantity of consumption goods these bequests effectively afford (i.e., after taking out consumption tax payments). Crucially, whenever $\varphi < \sigma$, the marginal utility of bequests falls more slowly than that of consumption as asset wealth increases, generating asset wealth over consumption ratios that are rising with asset wealth — consistent with the distributions of wealth (as well as bequeathed wealth) in the data. Finally, because our economy incorporates an exogenous productivity trend Γ_t, while our nonhomothetic preferences differ from balanced-growth preferences a la King et al. (1988), the scaling factors $\Gamma_t^{\varphi-\sigma}$ before the second term in the RHS of (2) is needed to ensure balanced growth.[7]

A *liquidity* motive, in addition to the bequest motive, becomes operative whenever $\alpha > 0$. In such a situation, households derive extra utility for holding government bonds relative to capital claims, so that the two assets cease paying the same equilibrium rate of return. This convenience yield is necessary to reconcile the low fiscal cost of public debt compared to the relatively high return on capital, both of which play an important role in our calibration. Absent the convenience yield, returns would be equalised across assets, and we would either misrepresent the government's true fiscal capacity (by overinflating the cost of public debt), or underestimate the gross return to capital, or both. The way we specify the utility from government bonds in (2) is guided by tractability: as we explain further in Section 3, this specification implies that in equilibrium (i) the return on government bonds is a constant fraction $1-\alpha \in (0,1]$ of the return to capital, allowing us to directly parameterise the preference parameter α from the cross-section of asset returns, and (ii) conditional on the convenience yield, households are indifferent about the share of each asset in their portfolio.

The last term in (2) reflects the disutility from working, which depends on the individual's age j and skill group ω as explained above, while $\Gamma_t^{1-\sigma}$ is here for balanced growth.

Constraints. In every period, the household faces the following budget constraint:

$$\begin{aligned}\left(1+\tau_t^C\right)c + b' + k' = \left(1+r_{b,t}\right)\left(b+\xi_t^b(\omega,j)\right) + \left(1+r_{k,t}\right)\left(k+\xi_t^k(\omega,j)\right) \\ + \mathbb{I}_{o=E}\left[\bar{h}_j e_j \omega z w_t - \mathcal{T}_t(\bar{h}_j e_j \omega z w_t)\right] + \mathrm{tr}_t(\omega,j,o) \qquad (3)\end{aligned}$$

In equation (3), $1+r_{b,t}$ and $1+r_{k,t}$ denote respectively the (after tax) returns to government bonds b and capital claims k carried over from the previous period. In addition to past portfolio choices (b,k), beginning-of-period bonds and capital claims incorporate the bequests (ξ_t^b, ξ_t^k) left by non-surviving households. Those bequests are received at the end of the previous period (hence they pay interest), and we index them to the recipient's individual state (ω, j) to accommodate the empirical distribution of bequests received across age and skill groups.

[7] The presence of $\underline{a}\Gamma_t > 0$ in the second term of the RHS of (2) bounds the marginal utility of assets away from infinity, making it possible that poor households make portfolio choices consistent with an occasionally binding borrowing constraint.

Next, active households (for whom $o = E$) receive gross labour earnings $\bar{h}_j e_j \omega w_t$ that depend on their individual productivity $\omega e_j z$, the number of hours they work $\bar{h}_j$, and the wage per effective hour w_t. Note that the household's age j affects both individual productivity e_j and the number of hours worked, consistent with the data (see Section 4 below). Households pay labour income taxes $\mathcal{T}_t(\bar{h}_j e_j \omega z w_t)$ on their gross earnings, where the tax function is parameterised to parsimoniously accommodate progressive taxation as in Benabou (2000) and Heathcote et al. (2017). More specifically, we assume that $\mathcal{T}_t(\cdot)$ is such that post-tax earnings are:

$$\bar{h}_j e_j \omega z w_t - \mathcal{T}_t(\bar{h}_j e_j \omega z w_t) = \left(1 - \tau_t^N\right) \left(\bar{h}_j w_t e_j \omega z\right)^{1-\zeta_t} \Gamma_t^{\zeta_t}, \tag{4}$$

where τ_t^N and ζ_t (both $\in [0,1)$) respectively denote the *level* and *progressivity* parameters of the labour income tax scheme. The correction $\Gamma_t^{\zeta_t}$ is needed for balanced growth whenever the tax code involves some progressivity.[8]

The last term in (3) is a transfer received from the government. One part of it (denoted $T_t > 0$ and implicit in (3)) consists of non-means-tested transfers, assumed to be paid lump-sum to all households in the economy. The other component is means-tested and is accordingly paid out only to inactive households ($o = I$). Among the latter, those who have not yet reached the legal retirement age ($j < J^R$) receive a basic universal income equal to a fraction ϕ^I of the effective unit wage w_t. Those who have reached the legal retirement age ($j \geq J^R$) receive a pension equal to a fraction ϕ^R of the average pre-tax wage of their own skill group, i.e., $\phi^R \omega \frac{\sum_{j \in J} e_j}{J-18} w_t$

Last, we assume that households cannot issue assets, so that:

$$b', k' \geq 0.$$

Individual problem. We are now in a position to set up the decision problem of the household. The individual state upon which decisions are based consists of the skill group ω, age j, stochastic productivity z, beginning-of-period portfolio (b, k) and last period's occupational status, o_{-}. It is convenient to separate households' decisions into two stages, namely a *labour-market participation* decision ($o \in \{E, I\}$) followed by *consumption and portfolio* decisions ($c, b', k' \geq 0$). Let V_t denote the household's maximal intertemporal utility at the time of the participation decision and W_t the intertemporal utility once the decision is made. In the first stage, the household solves the following binary choice problem:

$$V_t\left(\omega, j, z, b, k, o_{-}, \bar{\chi}\right) = \max\left\{W_t(\omega, j, z, b, k; o = E) + \bar{\chi}(E)\,,\ W_t(\omega, j, z, b, k; o = I) + \bar{\chi}(I)\right\}, \tag{5}$$

where the choice-specific preference shocks $\bar{\chi}(E), \bar{\chi}(I)$ are the product of i.i.d. Gumbel-distributed shocks χ and a balanced-growth correction, i.e., $\bar{\chi}(o) = \chi(o)\Gamma_t^{1-\sigma}$.

In the second stage, the household chooses consumption and assets conditional on current par-

[8] Intuitively, under progressive taxation and positive earnings growth, labour taxes eventually overwhelm labour earnings in the absence of a downward drift in the "effective" level parameter of the taxation scheme, i.e. $\left(1 - \tau_t^N\right)\Gamma_t^{\zeta_t}$.

ticipation and the distributions of next-period shocks to survival (ψ_{j+1}), productivity (z'), and preferences ($\bar{\chi}'$):

$$W_t(\omega, j, z, b, k; o) = \max_{c,b',k'\geq 0} \left\{ u(c, \omega, j, b', k', o) + \beta\psi_{j+1}\mathbb{E}_{z',\bar{\chi}'} V_{t+1}(\omega, j+1, z', b', k', o, \bar{\chi}') \right\}, \tag{6}$$

subject to (2), where $\beta \geq 0$ is the subjective discount factor, assumed constant and common to all households.

The problem differs slightly for households having reached the legal retirement age, since for them the choice of inactivity is irreversible. For an inactive retiree ($j \geq J^R \wedge o\_ = I$), the second-stage problem is as in (6) but the first-stage problem does not involve any choice and gives $V_t(\cdot) = W_t(\omega, j, z, b, k; o = I) + \bar{\chi}_t(I)$.

2.3 Firms

A representative firm produces a homogeneous final good, used for investment and public and private consumption alike, out of capital K_{t-1} and total effective hours worked, N_t, according to the technology

$$Y_t = ZK_{t-1}^{\theta}(\Gamma_t N_t)^{1-\theta}, \quad 0 \leq \theta \leq 1, \tag{7}$$

where Z is a constant and Γ_t a deterministic labour-augmenting productivity trend evolving as

$$\Gamma_t = (1+\gamma)\Gamma_{t-1}, \quad \gamma, \Gamma_0 > 0$$

Capital accumulates according to the law of motion

$$K_t = (1-\delta)K_{t-1} + I_t, \quad 0 \leq \delta \leq 1, \tag{8}$$

where I_t denotes investment.

It is important to note that, in our economy, measured TFP is endogenous due to the self-selection into activity of households with heterogeneous individual productivities. If one denotes by H_t the *observed* (as opposed to *effective*) total number of hours worked, then average labour efficiency is N_t / H_t while measured total factor productivity is

$$TFP_t = \frac{Y_t}{K_{t-1}^{\theta} H_t^{1-\theta}} = Z\Gamma_t^{1-\theta} \left(\frac{N_t}{H_t}\right)^{1-\theta}$$

Finally, since input markets are competitive, firms adjust their factor demands until marginal products equal factor prices:

$$\theta \frac{Y_t}{K_{t-1}} = (1+\tau_t^K) r_{k,t} + \delta, \quad (1-\theta)\frac{Y_t}{N_t} = w_t, \tag{9}$$

where τ_t^K is the corporate tax rate.

Notice that in our specification, the tax wedge between the pre-tax return to capital $(\theta Y_t / K_{t-1} - \delta)$ and the post-tax net return accruing to households $(r_{k,t})$ falls entirely on the corporate tax. Equivalently, we could have assumed that it is entirely borne by a linear capital income tax (paid by households rather than firms), or any combination of the two. Under the assumption of linearity, these alternative implementations of capital taxation are equivalent at any given level of tax revenue. In Section 4 below, we will calibrate the capital tax to match the overall revenue from capital taxation, and whether it is paid by households or firms in the model is immaterial.

2.4 Government

The flow budget constraint of the government is given by:

$$G_t + \mathrm{Tr}_t + (1 + r_t^b) B_{t-1} = \tau_t^C C_t + \mathcal{T}_t^N + \tau_t^K r_t^k K_{t-1} + B_t, \tag{10}$$

where G_t is government spending, Tr_t total transfers to households (whether means-tested or unconditional), B_{t-1} the stock of public debt inherited from the previous period, $\tau_t^C C_t$ the consumption tax revenue (where C_t denotes aggregate consumption), $\tau_t^K r_t^k K_{t-1}$ the capital tax revenue, and $\mathcal{T}_t^N$ the total revenue from labour taxation (i.e., the sum of all the $\mathcal{T}_t(\cdot)$ in equation (3)). We defer until Section 4 the discussion of the time path of all the variables in equation (10).

3 Equilibrium

3.1 Market clearing

Asset markets. We have assumed in Section 2.2 that households enjoyed a convenience yield from holding government bonds relative to capital claims. In general, the presence of such a convenience yield should imply that government bonds and capital are not substitutes, so that the two markets should clear separately. However, we show in Appendix A.2.2 that, under our specification for the flow utility functional (2), the Euler conditions for government bonds and capital claims are co-linear and yield the following necessary condition for optimality:

$$1 + r_{t+1}^b = (1 - \alpha)\left(1 + r_{t+1}^k\right), \tag{11}$$

where $1 - \alpha \in (0, 1]$. The latter condition is a no-arbitrage condition, whose violation would cause the demand for one of the assets to drop to zero, thereby restoring the equality. In the absence of a convenience yield ($\alpha = 0$), the two assets are perfect substitutes, and households are trivially indifferent between the two. In the presence of a convenience yield, households remain indifferent *conditional on the premium earned on capital claims.* That is, the utility benefit of holding government bonds is exactly offset by their lower returns across the entire asset demand curve.

Given this indifference, individual savings are well defined, but the exact split across portfolio shares is not and remains arbitrary. Accordingly, we may resolve this indeterminacy by imposing the condition that every household holds units of the market portfolio (B_t, K_t). Namely, defining a household's total savings as $a' \equiv k' + b'$, we assume that

$$k' = \left(\frac{K_t}{K_t + B_t}\right) a' \quad \text{and} \quad b' = \left(\frac{B_t}{K_t + B_t}\right) a' \tag{12}$$

With these portfolio shares, portfolio returns are symmetric across households and given by

$$1 + r_t = \frac{(1 + r_{b,t})b + (1 + r_{k,t})k}{a} = (1 + r_{k,t})(1 - \Upsilon_{t-1}), \tag{13}$$

where $\Upsilon_t = \alpha B_t / (K_t + B_t)$ (an aggregate variable) is a time-varying discount on the return to capital $1 + r_{k,t}$ reflecting the presence of lower-return government bonds in household portfolios. Last, substituting equations (11) to (13) into the household budget constraint (3), and exploiting the fact that the assets received as bequests are in similar proportions as one's own savings (by construction), we can simplify the budget constraint (3) into:

$$\left(1 + \tau_t^C\right) c + a' = (1 + r_t)\left(a + \xi_t(\omega, j)\right) + \mathbb{I}_{o=E}\left(1 - \tau_t^N\right)\left(\bar{h}_j w_t e_j \omega z\right)^{1-\zeta_t} \Gamma_t^{\zeta_t} + \mathrm{tr}_t(\omega, j, o), \tag{14}$$

where $\xi_t(\omega, j) = (B_{t-1} + K_{t-1})\xi_t^k(\omega, j)/K_{t-1}$. Similarly, we can use (12) and the definition of Υ_t to replace (b', k') by a' in equations (2) and (6), thereby reducing individual savings choice to a single variable (rather than having to keep track of b and k separately).[9] Eventually, aggregating time t saving choices a' across all surviving households (including migrants) into A_t, and then adding total bequests Ξ_t to households' savings, asset market-clearing at time t can be summarised by a single condition, namely:

$$A_t + \Xi_t = K_t + B_t \tag{15}$$

Labour and goods markets. The labour market clears, so that the sum of effective labour supplies (adjusted for productivity and skill levels) equals the firms' demand for labour (see Appendix A.6 for details). By Walras law, the goods market clears whenever both the labour and asset markets do.

3.2 Outline of equilibrium definition

Given the economy's demographic structure and sequences of government policies, an equilibrium in this economy is given by: sequences of prices; sequences of households' values and associated decision rules; sequences of aggregate quantities; and sequences of measures of households across the state space such that (i) households' decision rules solve their individual problem and achieve the corresponding values; (ii) firms maximise profits; (iii) the government's budget constraint is satisfied;

[9] See Appendix A.2.3 for details.

(iv) labour, asset and goods markets clear; and (v) the measure of households evolves consistently with their decision rules. Appendix A.7 provides a full formal definition of this equilibrium.

4 Calibration

In this section, we calibrate (i) the economy's initial balanced-growth path (summarised in Table 1) and (ii) our baseline path of government spending and public debt used throughout the analysis.

4.1 Initial balanced growth path

4.1.1 Households

Demographics and wealth transmission. The population shares μ_j and survival rates ψ_j, as well as population growth, n (= 0.5%), are from the UN's 2024 World Population Prospects. The migrants to natives ratios, $M_{j,t}/L_{j,t}$, are treated as the residuals of equation (1).

Households leave accidental bequests upon death, which are distributed to beneficiaries according to their age j and skill group ω. To be more specific, we decompose received bequests in equation (3) as $\xi_t(\omega, j) = \xi_t \xi_\omega \xi_j$, where (ξ_ω, ξ_j) are age and group-specific constants and ξ_t is a time-varying intercept ensuring equality between the collection of bequests from the deceased and its distribution to survivors (ξ_t is constant in any balanced-growth path but not along the transition between them). We calibrate the ξ_ωs by assuming that bequests are redistributed proportionally to beneficiaries' skill group ω (e.g., $\xi_{\omega_1}/\xi_{\omega_2} = \omega_1/\omega_2$, where the ωs are discussed below), reflecting the fact that high-income individuals also inherit more wealth. The ξ_js are calibrated to reproduce the age distribution of received bequests and, absent a specific data source for France, we impute the distribution obtained from the US Survey of Consumer Finances as computed by Osorio and Huntley (2021).

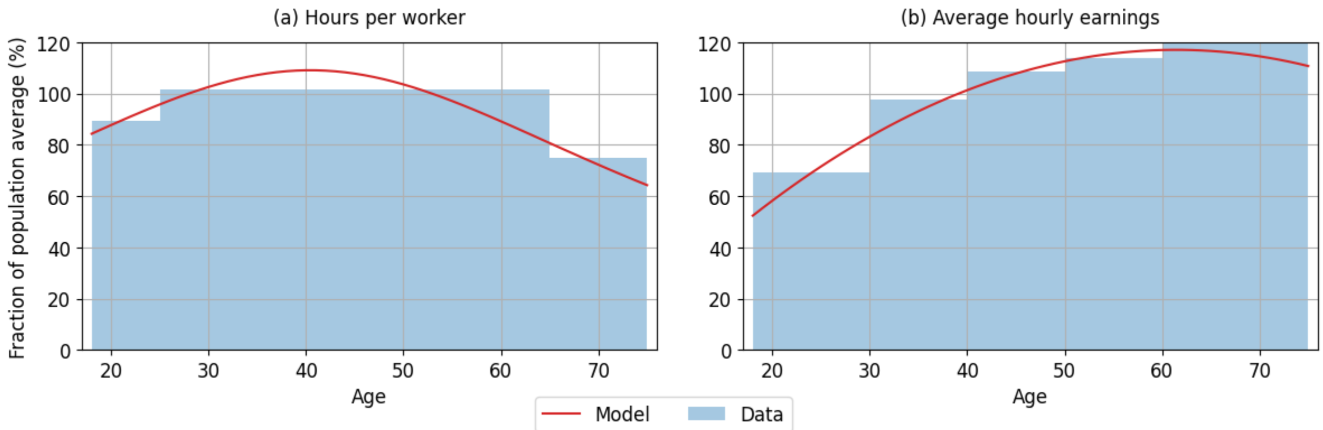
Effective hours of work. An active individual provides $\bar{h}_j \times \omega \times e_j \times z$ effective hours of work to the representative firm. For hours worked by age $\bar{h}_j$, we first normalise the working time units such that the average number of hours worked per worker is 1, and then fit a quadratic trend to (the inverse of) the hours worked by broad age groups of the European Union Labour Force Survey (EU-LFS) to interpolate the hours worked for every age group in the model (Figure 2, Panel (a)). We proceed similarly for the age-specific productivities e_j, normalising to 1 the average productivity of individuals aged 18-29, and then fitting a quadratic trend to EU-LFS hourly wages by broad age group to interpolate age-specific productivities for every age in the model (Figure 2, Panel (b)).

Next, we calibrate the levels and distributions of permanent productivity levels (ω) as follows. For international comparisons, Eurostat aggregates education levels into three groups of the International Standard Classification of Education, namely ISCED 0-2 (lower secondary education or less), ISCED 3-4 (more than 0-2 and non-tertiary education) and ISCED 5-8 (more than 0-4 and up to doctoral degree). We equate the probability of each draw at "birth" $\pi^\Omega(\omega)$ to the empirical masses of individuals in these broad educational groups. We then use data on mean gross hourly earnings by education

Table 1: Calibration of initial balanced-growth path.

Parameter	Description	Value	Source/Target
Demographics and wealth transmission			
μ_j	Population shares		UN 2024 World Population Prospects
ψ_j	Survival rates		UN 2024 World Population Prospects
n	Population growth	0.5 %	UN 2024 World Population Prospects
ξ_ω	Bequests by skill group		Proportional to skill group ω
ξ_j	Bequests by age group		Osorio and Huntley (2021)
Effective hours of work			
$\bar{h}_j$	Hours by age group	Fig. 2	Fitted hours from EU Labour Force Survey
e_j	Productivity by age group	Fig. 2	Fitted earnings from Struct. Earnings Survey
ω	Permanent prod. (levels)	Tab. 2	Structure of Earnings Survey
$\pi^{\mathbf{\Omega}}(\omega)$	Permanent prod. (shares)	Tab. 2	Eurostat
z	Transitory productivity levels		Imputed from GRID
Π^Z	Transition matrix across z		Imputed from GRID
Preferences			
β	Subjective discount factor	0.99	Wealth/GDP = 4.60 (Eurostat)
σ	Cons. utility curvature	2	Standard
φ	Bequests utility curvature	1.5	Bequest distribution (Figure 3)
ν_a	Bequests scaling factor	2.6	Bequests/GDP = 9%
α	Liquidity preference	0.03	Real annual bond rate $r_b = 1.5\%$
$\kappa_{\omega,j}$	Disutility from working	Fig. 4	Participation by skill/age groups (Eurostat)
σ_χ	Scale of Gumbel shocks	0.015	Aggregate LPE = 1.08 (Erosa et al., 2016)
Firms			
$1-\theta$	Labour share	62.0%	Federal Reserve Economic Data (FRED)
γ	Labour productivity growth	0.87%	2004-2019 average (Bergeaud et al., 2016)
δ	Depreciation rate	3.1%	Pretax rental rate of capital of 8%
Government: fiscal policy			
G/Y	Public consumption/GDP	24.1%	Eurostat
ζ	Labour tax progressivity	0.138	Malmberg (2025)
τ^N	Labour tax level parameter	34.5%	Rev./GDP = 22,7% (EC Taxation Trends)
$\tau^K/(1+\tau^K)$	Capital tax wedge	39.1%	Rev./GDP = 10,4% (EC Taxation Trends)
$\tau^C/(1+\tau^C)$	Consumption tax wedge	15.1%	Rev./GDP = 10,8% (EC Taxation Trends)
B/Y	Public debt/GDP	113.0%	Eurostat
T/Y	Lump-sum transfer/GDP	4.0%	Residual of government budget constraint
Government: social security			
J^R	Legal retirement age	63	French social-security regulation
ϕ^R	Pension replacement param.	53.6%	Pensions/GDP = 14,5% (EC Tax. Trends)
ϕ^I	Guaranteed income param.	22.2%	OECD Adequacy of Min. Income Benefits

Figure 2: Hours worked and average hourly earnings by age (%).



Notes: On Panel (a), the blue bars plot the "average usual weekly hours worked in the main job" by age groups (15-24, 25-64, 65-74) from the 2024 EU-Labour Force Survey and the red curve the model's fitted average hours worked by age for active individuals (the fitted curve is $h_j^{-1} = 1.19 - 0.024 \times (j - 18) + 0.0005 \times (j - 18)^2$). On Panel (b), the blue bars plot the average hourly earnings by age groups (20-29, 30-39, 40-49, 50-59, 60+) from the 2022 EU-Structure of Earnings Survey and the red curve the fitted age-specific productivities (the fitted curve is $e_j = 0.78 + 0.044 \times (j - 18) - 0.0005 \times (j - 18)^2$).

group from the EU Structure of Earnings Survey to recover relative permanent productivities between those education groups. Finally, we normalise permanent productivity levels such that average permanent productivity $\sum_{\omega \in \boldsymbol{\Omega}} \pi^{\boldsymbol{\Omega}}(\omega)\omega$ equals 1. The implied distribution of permanent productivity levels is summarised in Table 2.

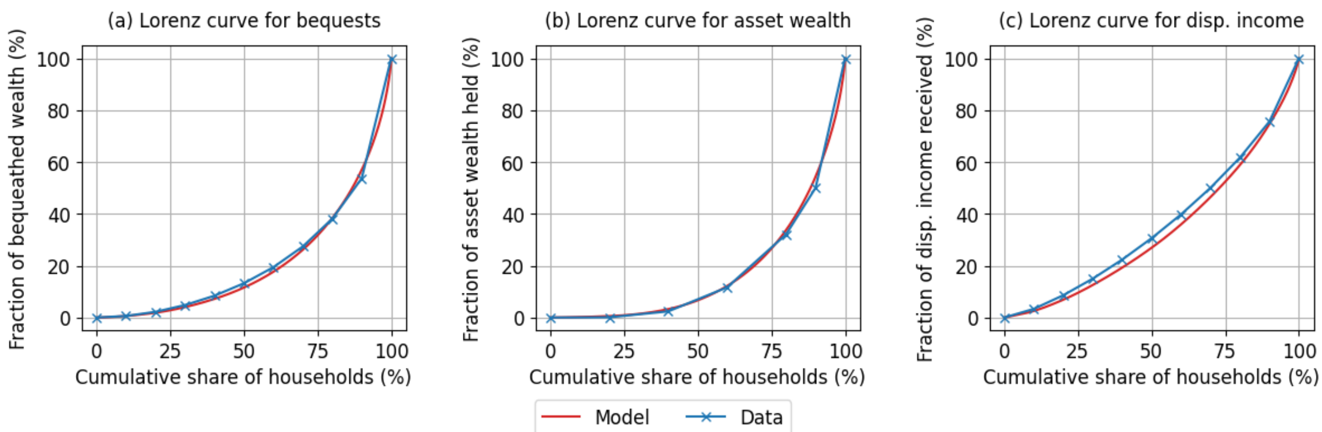
Last, we calibrate the stochastic process for transitory productivity (z) as follows. We assume that $\ln(z)$ follows an AR(1) process. We take the variance of log residual income in France (after controlling for age), measured by Kramarz et al. (2022) and used in GRID (Guvenen et al., 2022b) as a proxy for the cross-sectional variance of total (log) productivity excluding the age component, i.e. $\ln(\omega z)$. We then subtract the share of this cross-sectional variance explained by permanent-income differences (as calibrated in Table 2) and assign the residual variance to transitory idiosyncratic shocks. This identifies the unconditional variance of transitory productivity, but not the persistence and the variance of innovations separately. To recover the latter, the persistence of log-idiosyncratic productivity is calibrated such that the persistence of total log-productivity matches the (within-cohort) 5-year per-

Table 2: Distribution of permanent productivity ("skills").

	Distribution	
Education	Level[(a)]	Pop. share (%)
Lower-secondary education	0.76	19.0
Upper-secondary education	0.84	42.8
Tertiary education	1.29	38.2

[(a)] Permanent productivity levels are normalised such that average economy-wide permanent productivity is equal to 1.

Figure 3: Lorenz curves for bequeathed wealth, asset wealth, and disposable income.



Notes: The (targeted) empirical distribution of bequeathed wealth in Panel (a) is from Allegre (2007). The empirical distributions of wealth (Panel (b)) is from the 2021 wave of the ECB's Household Survey of Consumer Finances, and that of disposable income (Panel (c)) is from the 2021 wave of the French Fiscal and Social Incomes Survey (INSEE, 2024).

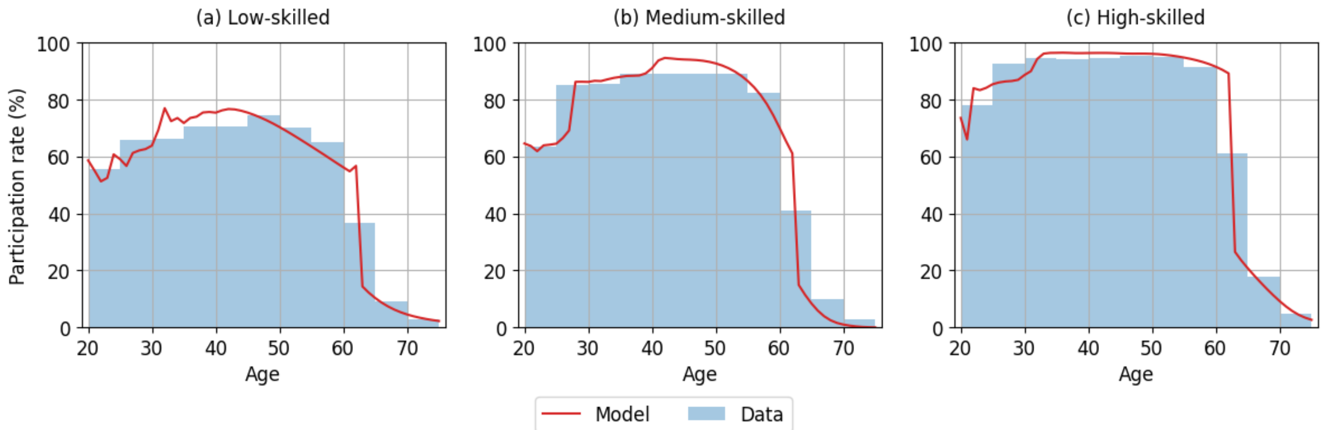
sistence of income rank estimated by Guvenen et al. (2022b) for France over the 1997-2007 period. Finally, we use the Rouwenhorst method to discretise the resulting AR(1) process as a Markov chain with $N_z = 7$ states.

Preferences. Households' subjective discount factor is set to $\beta = 0.99$ to match a total asset over GDP ratio A/Y of 460%. This corresponds to a public debt-to-GDP ratio B/Y of 113% and a fixed capital-to-GDP ratio K/Y of 347%, both from Eurostat in 2024. We set the CRRA parameter to $\sigma = 2$. The bequest curvature parameter $\varphi = 1.5 < \sigma$ is set to match the distribution of bequeathed wealth in Allegre (2007). As shown in Figure 3, Panels (a) and (b), the model's implied bequest and wealth distributions closely match their empirical counterparts. The scaling parameter ν_a is set to match a bequest-to-GDP ratio Ξ/Y of 9%.[10] The convenience yield parameter α is set to produce an annual real interest rate on public debt of $r_b = 1.5\%$ annually, given the return to capital claims r_k (calibrated below).

Households' decisions to participate in the labour market is affected by their disutility from labour supply, which contains two components, namely the skill/age constant $\kappa_{\omega,j}$, $\omega \in \boldsymbol{\Omega}$, $j \in \boldsymbol{J}$ in equation (2), and the choice-specific shocks $\chi(o)$, $o \in \{I, E\}$ in equation (5). We use Eurostat data on participation levels by education and age groups to calibrate the former. More specifically, for each of the three educational groups under consideration, we fit a group-specific quadratic age-trend $\kappa_{\omega,j} = a_\omega + b_\omega(j-18) + c_\omega(j-18)^2$ to match labour-market participation by broad age groups; this allows interpolating participation disutility and implied participation rates for every age and education group in our model (Figure 4). Regarding the choice-specific shocks, we set the scale parameter of the Gumbel distribution to match an aggregate labour participation elasticity to a one-off wage shock of 1.08

[10]According to Piketty and Zucman (2015) and Dherbécourt et al. (2021), intergenerational wealth transfers (whether bequests or *inter vivos* gifts) amount to 15% of French national income. Besides, Dherbécourt (2017) documents that bequests proper represent 60% of total intergenerational wealth transfers in France.

Figure 4: Labour force participation rates by age and skills (%).



Notes: The blue bars plot the average labour force participation rates by educational level from the 2024 EU-Labour Force Survey. The red curve plots the model's participation rates by age given the fitted distribution of participation disutilities $\kappa_{\omega,j}$ (for the low-skilled, $\kappa_{\omega,j} = 1.58 - 0.01 \times (j-18) - 0.0001 \times (j-18)^2$; for the medium-skilled $\kappa_{\omega,j} = 1.63 - 0.055 \times (j-18) + 0.0007 \times (j-18)^2$; and for the high-skilled, $\kappa_{\omega,j} = 1.50 - 0.046 \times (j-18) + 0.0004 \times (j-18)^2$).

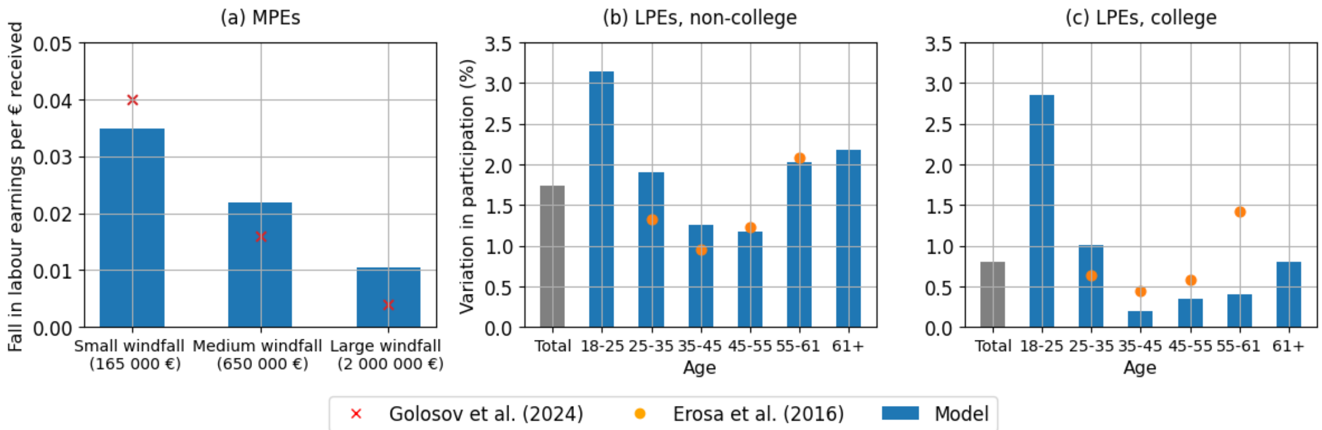
among prime-age households (25-61), following Erosa et al. (2016). Given this aggregate LPE, our calibrated $\kappa_{\omega,j}$s generate a distribution of age- and education-specific LPEs. As expected, and shown in Panels (b) and (c) of Figure 5 (where the first two skill groups of Table 2 are aggregated into "non-college" for conciseness), LPEs fall with education at any given age and are U-shaped in age within education groups. For comparison, we also report the distribution of LPEs implied by Erosa et al. (2016)'s structural model. The two models differ in both features and calibration (in particular, the other model uses a narrower age window than ours), yet the implied LPEs are broadly consistent. The main difference is our more conservative LPE for colleged-educated indivdiuals aged 55-61.

LPEs to a one-off wage change reflect households' willingness to engage in intertemporal substitution in labour supply. We also check that our calibrated preferences deliver reasonable intertemporal income effects by computing households' Marginal Propensities to Earn (MPEs), namely, the reduction in labour earnings associated with a windfall unearned gain. As Panel (a) of Figure 5 shows, aggregate MPEs in the model align well with those recently evidenced by Golosov et al. (2024) for windfall gains of various sizes. Overall, Figure 5 makes us confident that the model has reasonable implications about the relative strengths of substitution versus income effects on labour supply, both of which play a key role in the analysis that follows.

4.1.2 Firms

The parameter θ in the production function (7) is calibrated to match a labour share of 62% (from FRED). Productivity growth γ is set to 0.87% per year, the historical average over 2004-2019 according to the Long Term Productivity database of Bergeaud et al. (2016). We calibrate the annual pretax rental rate of capital $(1+\tau_t^K)r_{k,t}$ to 8%, consistent with the evidence in Marx et al. (2021) and Jordà et al. (2019) for the euro area. Given this rental rate, the depreciation rate consistent with a fixed-capital-to-output

Figure 5: Marginal Propensities to Earn and Labour Participation Elasticities.



Notes: Panel (a) plots the model-implied marginal propensities to earn (MPEs) out of unearned income, that is, the immediate fall in post-tax labour-market earnings following a one-off windfall gain, for different gain sizes (165 000 €, 650 000 € and 2 000 000 €). This is compared to the corresponding estimates from Golosov et al. (2024), obtained by multiplying the overall MPE estimates of their Figure B.6.(a) with the extensive-margin share reported in their Table A.3. Panels (b)-(c) plot the model-implied distribution of labour participation elasticities (LPEs), that is, the immediate proportional response of labour market participation after a one-off 1% increase of the real wage, holding everything else unchanged. Low- and medium-skilled individuals are pooled into "non-college" for conciseness. This is compared with the corresponding distribution of LPEs in Erosa et al. (2016) for the age groups common to both models.

ratio of $K/Y = 3.47$ (discussed above) and $\theta = 38\%$ is $\delta = 3.1\%$ annually (by equation (9)). $Z = 0.64$ is a normalisation constant, which we set so that the initial (detrended) output is 1.

4.1.3 Government.

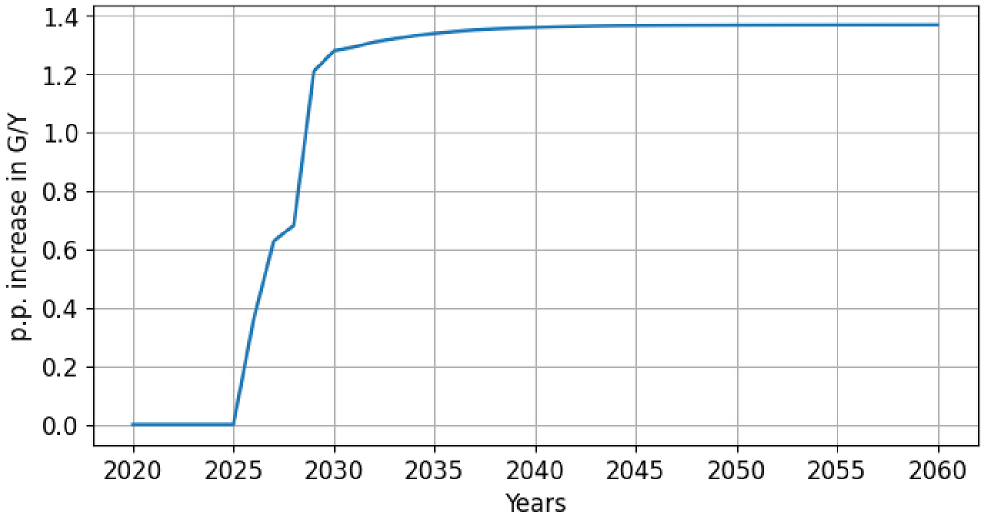
Social security. We set the legal retirement age (before any policy change) to $J^R = 63$ (the current legal age in France). As explained in Section 2.2 above, all individuals belonging to skill group $\omega \in \mathbf{\Omega}$ get the retirement benefit $\phi^R \omega \frac{\sum_{j \in J} e_j}{J-18} w_t$. The parameter ϕ^R is then set to match the pension-to-GDP ratio reported by the European Commission Data on Taxation Trends. A household that is inactive but not yet retired receives a transfer $\phi^I w_t$. ϕ^I is calibrated such that the average disposable non-financial income of these households equals 30.3% of the population's average disposable non-financial income, consistent with the OECD's 2024 benchmark for the adequacy of minimum income benefits.[11]

Taxes. We set the progressivity parameter of the labour income tax scheme to $\zeta = 0.138$, as estimated by Malmberg (2025) for the French economy. The level parameter of the scheme is then set to $\tau^N = 0.345$; this yields an overall revenue from labour taxation of 22,7% of GDP, consistent with European Commission Data on Taxation Trends for the French economy in 2024.[12] The (linear) taxes on

[11] We exclude financial revenues because all non-participating non-retired households receive this transfer regardless of their asset wealth. As a result, some recipients may be very wealthy and therefore receive large financial revenues. Such households are likely not included in the OECD's indicator.

[12] In the model as well as in the data, the revenue from labour taxation is inclusive of all social contributions.

Figure 6: Benchmark path of government spending to-GDP ratio.



Notes: The curve plots the path of the government spending-to-GDP ratio (G/Y) after a gradual shift in the legal retirement age, holding all other fiscal instruments (including the debt-to-GDP ratio) unchanged. The legal retirement age increases from 63 to 64 in 2028, then from 64 to 65 in 2030. G/Y starts rising at the time of the announcement (2026), due to anticipatory effects on labour supply. The policy increases G/Y by 1.28 pp. in 2030 and by 1.37 pp. in the long run.

consumption and capital are similarly calibrated to match the corresponding revenue-to-GDP ratios in the 2024 EC Taxation Trends. For the consumption tax, this ratio was 10.8%; this implies a consumption tax rate of $\tau^C = 17.8\%$ and a consumption tax *wedge* (the fraction of consumption expenditures $(1+\tau^C)C$ accruing to the government) of $\tau^C/(1+\tau^C) = 15.1\%$. On the other hand, the capital tax revenue-to-GDP ratio was 10.4% in France in 2024, which implies a capital tax wedge (i.e., the fraction of the pre-tax return to capital $\theta Y/K_{-1} - \delta$ accruing to the government) of $\tau^K/(1+\tau^K) = 39.1\%$. As discussed in Section 2.3, any combination of linear corporate and capital-income taxes yielding the same overall revenue from capital taxation would be equivalent. The lump-sum transfer T_t, which is part of total transfers Tr_t in equation (10), is treated as a residual that clears the government budget constraint, given government spending and all other tax parameters at the initial balanced-growth path ("BGP" henceforth); the resulting value is 4.0% of real GDP.

Interestingly, given our calibration for effective hours of work as well as the tax-and-transfer and social-security systems, the model's implied disposable income distribution closely matches the (untargeted) distribution of disposable income in France, as shown on Panel (c) of Figure 3.

4.2 Benchmark fiscal policy

Having fully described the initial BGP of the economy, we now characterise our benchmark policy paths for public debt and government spending (both as ratios of GDP), which will be kept unchanged across all alternative fiscal adjustments considered below. First, we assume that the public debt-to-GDP ratio remains constant and equal to its value at the initial GDP after the government spending

Table 3: Macroeconomic effects.
(% change from initial to final balanced-growth paths)[(a)]

	Y	K/Y	TFP	$Emp.$	C
1 instrument					
Labor tax (level)	−2.9	−3.1	0.6	−2.0	−4.3
Labor tax (progressivity)	−2.2	−4.0	−0.5	1.1	−3.4
Capital tax	−1.6	−4.4	−0.5	2.1	−2.7
Consumption tax	0.2	−0.6	−0.3	1.2	−1.9
Retirement age +2y	2.2	−0.5	0.1	2.8	0.1
Pensions	1.5	0.5	0.2	1.1	−1.0
2 instruments: retirement age +1y and					
Labor income tax (level)	−0.2	−1.7	0.3	0.6	−2.0
Labor income tax (progressivity)	0.1	−2.2	−0.2	2.1	−1.6
Capital tax	0.4	−2.4	−0.2	2.5	−1.3
Consumption tax	1.3	−0.5	−0.1	2.1	−0.9
Pensions	1.9	0.0	0.1	2.0	−0.5

[(a)]The table shows, for each fiscal adjustment under consideration (Column 1), the induced proportional difference in macroeconomic aggregates between the initial and final balanced-growth paths.

shock has occurred.[13] Next, we take as our benchmark path of government spending-over-GDP that generated by an increase in the legal retirement age J^R by two years (from age 63 to 65), *holding all other fiscal instruments unchanged at their initial values.* Intuitively, this answers the question, "How much extra spending could the government afford if it were to shift the legal retirement age by two years?" In the long run, this reform changes the government spending-to-GDP ratio by 1.37%, close to the NATO guidelines change and in the ballpark of current policy discussions. Assuming that the shift is first announced (in 2026) and then gradually implemented (from 2027 onwards) by increments of six months every year (in such the legal retirement age becomes 64 in 2028 and 65 in 2030), this produces a gradual increase in G/Y plateauing after a couple of decades, as shown in Figure 6.

5 Aggregate and distributional effects of alternative fiscal adjustments

We are now in a position to conduct our main policy experiments. For expositional clarity, we consider a limited set of potential financing scenarios and proceed as follows. We first consider fiscal

[13]We could easily allow for alternative paths of the public debt-to-GDP ratio, but deliberately refrain from doing so for three reasons. First, tax-smoothing arguments suggest a limited role for public debt in offsetting a *permanent* spending shock like that under consideration. Second, an upward shift in the long-run value of the debt-to-GDP ratio could only *increase* the tax burden at that horizon, thereby further reducing fiscal space rather than expanding it. Last, while we take due note of Marzian and Trebesch (2026)'s observation that many past peacetime buildups have been partly debt-financed, this is likely not an option for high-debt countries like France, on which our calibration is based.

Table 4: Changes in fiscal revenue.
(pp. change from initial to final balanced-growth paths)[(a)]

	$\frac{\mathcal{T}^N}{Y}$	$\frac{\tau^K r^k K}{Y}$	$\frac{\tau^C C}{Y}$	$-\frac{\text{Tr}}{Y}$	$-\frac{r_b B}{Y}$
1 instrument					
Labor tax (level)	**2.2**	0.1	−0.2	−0.5	−0.2
Labor tax (progressivity)	**1.7**	0.2	−0.1	−0.0	−0.3
Capital tax	−0.2	**1.3**	−0.1	0.3	0.0
Consumption tax	−0.0	0.0	**1.2**	0.2	−0.0
Retirement age +2y	−0.0	0.0	−0.2	**1.7**	−0.0
Pensions	0.0	−0.0	−0.3	**1.6**	0.0
2 instruments: retirement age +1y and					
Labor tax (level)	**1.0**	0.1	−0.2	**0.6**	−0.1
Labor tax (progressivity)	**0.8**	0.1	−0.2	**0.9**	−0.2
Capital tax	−0.1	**0.6**	−0.2	**1.0**	−0.0
Consumption tax	−0.0	0.0	**0.5**	**1.0**	−0.0
Pensions	−0.0	−0.0	−0.2	**1.6**	0.0

[(a)]The table shows, for each fiscal adjustment under consideration (Column 1), the percentage-point difference in fiscal revenue sources over GDP between the initial and the final balanced-growth paths. By the government budget constraint (10), same-row numbers (up to rounding) sum up to 1.37, i.e., the pp. difference in G/Y between the final and initial BGPs (by our time-index convention, B and K here are understood to be time-indexed $t-1$, while all other variables are timed t). Bolded numbers highlight which revenue source is most directly affected by the said fiscal adjustment.

adjustments involving *a single tax or social-security instrument.* This includes, for example, the two-year shift in the legal retirement age used to construct our benchmark government spending-to-GDP path in Figure 6. This also includes fiscal reforms in which the government spending shock is entirely financed by adjusting one of the parameters of the labour income tax scheme (the level or progressivity) or by adjusting another tax instrument (on capital or consumption). Because such experiments move exactly one fiscal instrument at a time, they provide clear insights into how each one affects the equilibrium. In a second step, we consider a set of mixed fiscal adjustments, all involving a one-year (rather than two-year) shift in the legal retirement age, plus another instrument to cover the resulting shortfall. This selection is guided by conciseness, but of course, any combination of instruments can be contemplated.

We assess the long-run macroeconomic, fiscal, and distributional effects of each adjustment by computing the difference between outcomes on the initial and final balanced-growth paths. Accordingly, Table 3 reports the proportional changes in output, inputs and consumption induced by the fiscal shock, thereby decomposing the "crowding out" of total private consumption (Column 6) into its proximate causes (Columns 3 to 5). Table 4 breaks down the burden of higher spending across revenue sources in the government budget constraint (10); this captures not only the direct impact of each (set of) instrument(s) being adjusted but also all induced behavioural and general-equilibrium responses. Finally, Figures 7-8 display the contribution of each age and skill group — that is, the proportional within-group change *times* the size of the group — to the overall contraction in employment and consumption; this reveals which households in the economy effectively drives the reported macro and fiscal responses. We defer until Section 6 the discussion of the tradeoff between aggregate consumption versus consumption inequality across fiscal packages.

5.1 Single-handed fiscal adjustments

5.1.1 Labour taxation

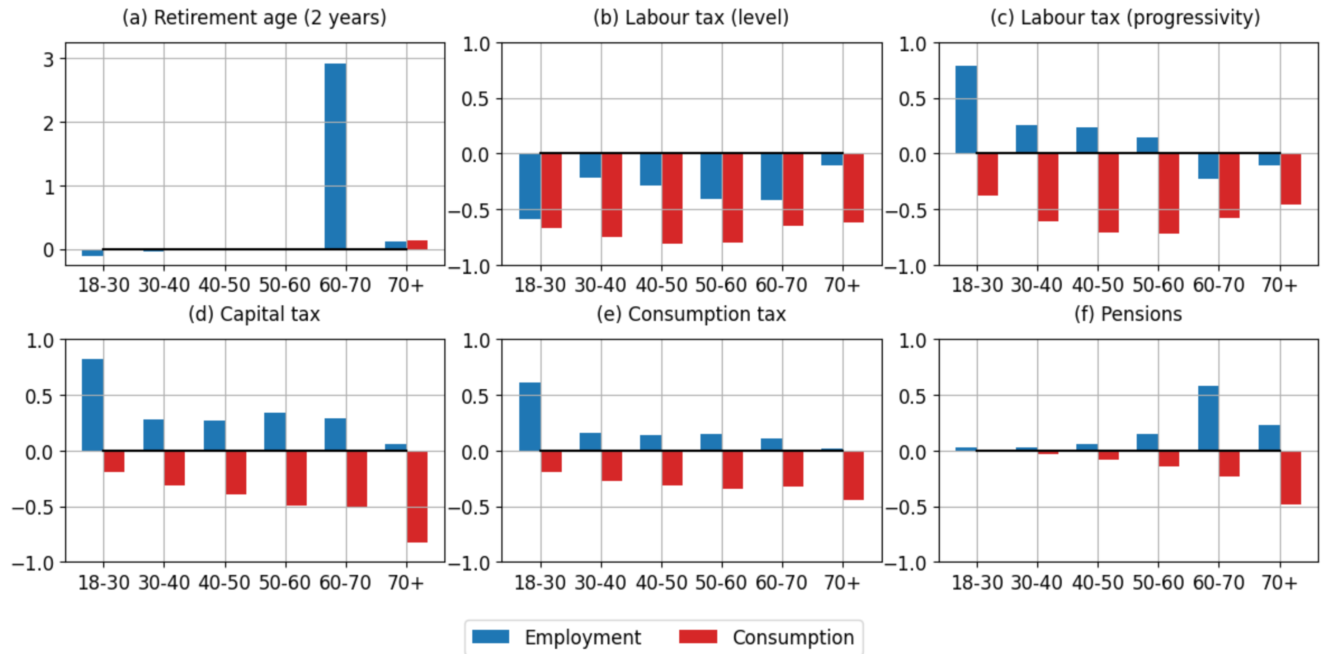
We begin by discussing changes to the labour tax schedule, whether in the level (τ_t^N) or progressivity (ζ_t) parameter of the tax schedule (4) that maps gross into net earnings. The second and third rows of Tables 3-4 show that the two labour-tax parameters elicit very different macro and fiscal responses — though both lead to sizable contractions in output and consumption.

Broad-based labour taxation. A broad-based increase in labour taxation is achieved by raising the *level* parameter of the tax schedule to the amount required by the extra government spending. This policy sharply contracts employment (fifth column of Table 3), because it reduces the relative pay-off from working and thereby pushes high-LPE households out of the labour force. Because those dropouts are the least productive workers (they were close to the margin of participation before the shock), the change in the composition of the workforce results in a mild increase in TFP (see equation (2.3) and the third column of 3). Yet the employment contraction largely dominates and eventually spills over into the accumulation of capital: on the one hand, the aggregate supply of savings falls as households' permanent income declines; on the other hand, the demand for capital falls due to its complementarity with labour input. Ultimately, increasing broad-based labour taxation to cover the fiscal shortfall is the most destructive policy for aggregate output and consumption.

These aggregate effects are best understood by disaggregating them into individual labour-supply and consumption responses. Except for the 70+ age bracket (whose members are typically out of the workforce regardless of the policy change), the employment responses by age group in Panel (b) of Figure 7 mirrors the distribution of LPEs in Figure 5: individuals aged 30 to 50 (who have relatively low LPEs) leave employment relatively less than those aged 18-30 or 50+. To the extent that individuals aged 30 to 50 are those relying most on labour earnings for income, their contribution to the fall in consumption is also greater than that of other groups. The pattern of LPEs in Figure 5 also implies that individuals with more skills are also relatively less likely to exit the labour force after the tax hike. However, medium-skilled individuals represent a large share of the population in the first place (48.2% according to Table 2), despite having lower LPEs than the low-skilled; this explains why they dominate the contribution to the employment fall by skill group in Panel (b) of Figure 8.

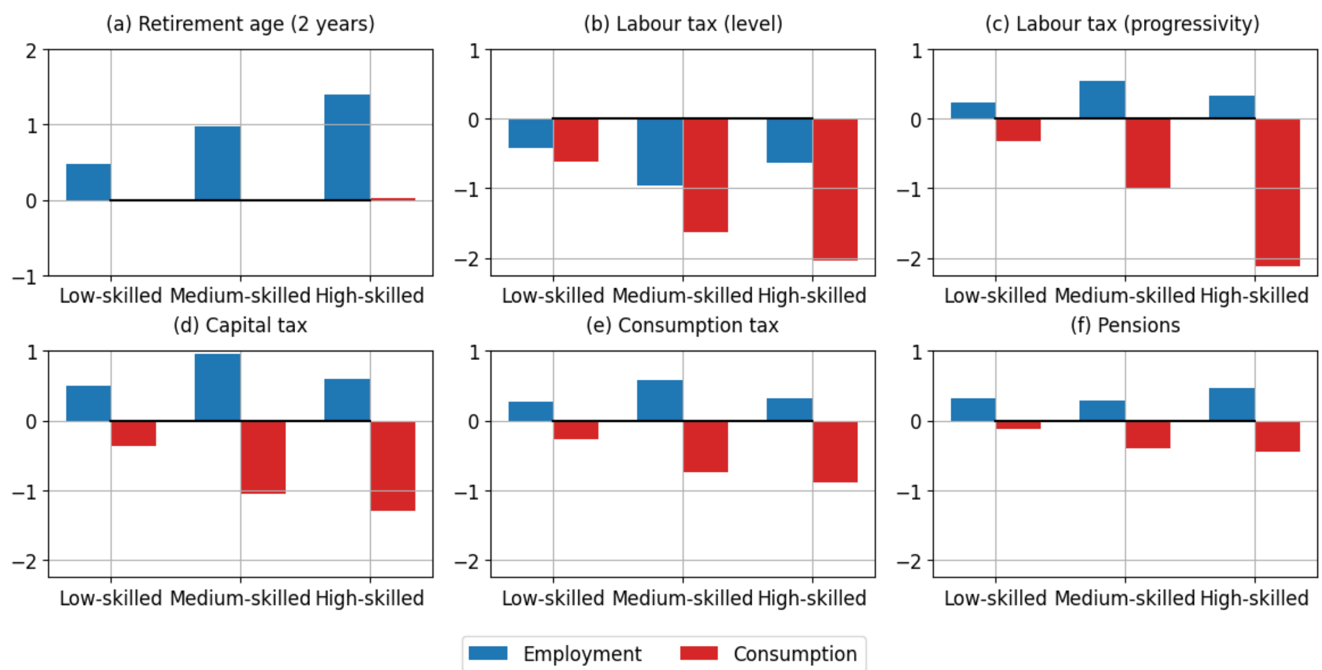
Taken together, these numbers suggest that the ability of broad-based taxation to efficiently raise revenue for additional government spending is limited. A direct way to diagnose the potential self-destructiveness of a tax is to construct the associated *Laffer curve*, which plots tax revenue across BGPs with different tax levels, accounting for all behavioural and general-equilibrium effects of taxation.

Figure 7: Contributions by age groups to the aggregate change (pp.) (single-handed adjustments)



Notes: Each bar represents the contribution (in pp.) of a given group to the aggregate change in the variable (in % deviation from the initial BGP). This is equal to the % change in group-level consumption and employment *times* the fraction of the population belonging to that group.

Figure 8: Contributions by skill groups to the aggregate change (pp.) (single-handed adjustments)



Notes: Each bar represents the contribution (in pp.) of a given group to the aggregate change in the variable (in % deviation from the initial BGP). This is equal to the % change in group-level consumption and employment *times* the fraction of the population belonging to that group.

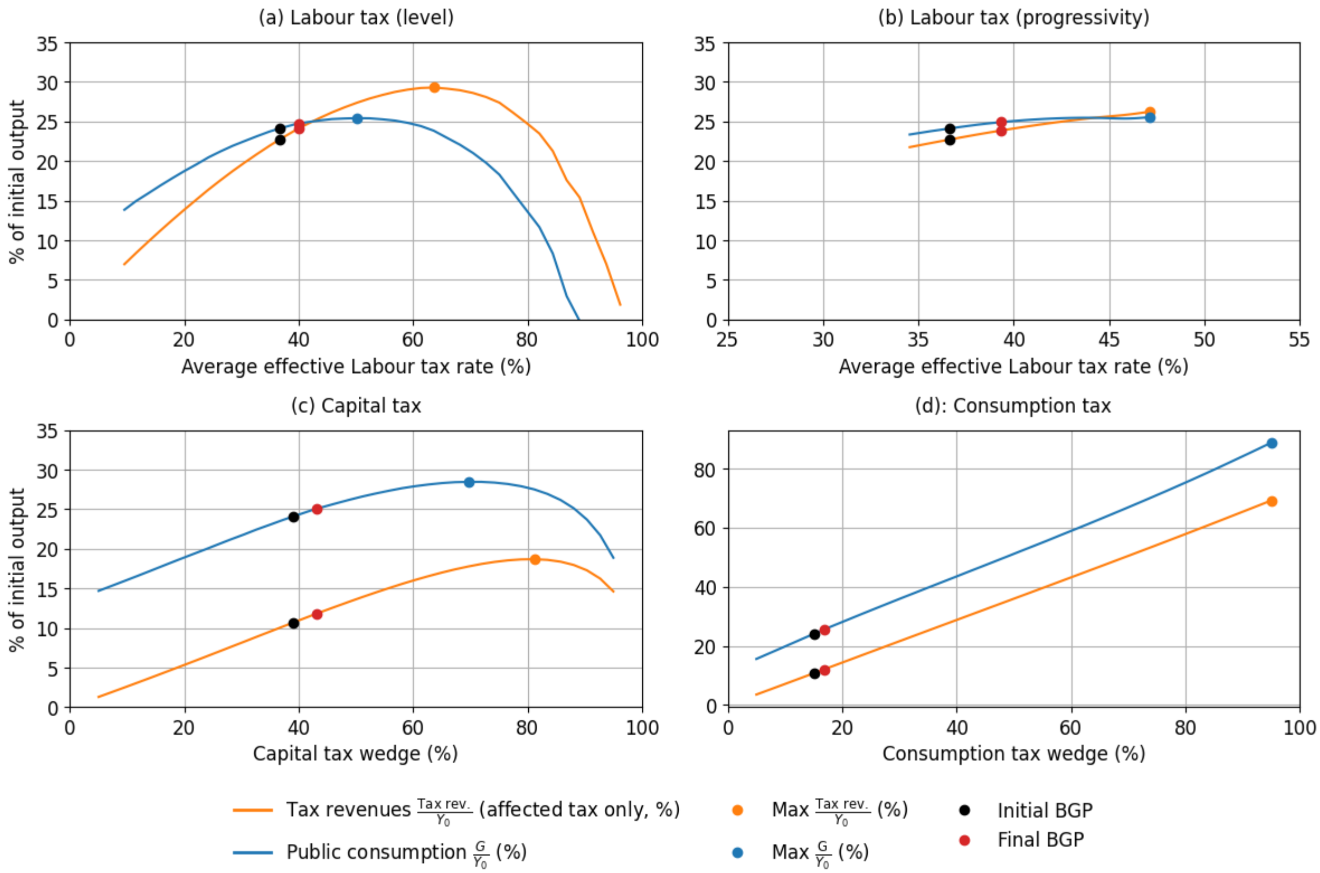
Accordingly, Panel (a) of Figure 9 shows two such curves when the level parameter of the labour tax schedule is adjusted. To clarify, the x-axis' scale displays the average effective labour tax rate (defined as the ratio of aggregate labour tax revenues to aggregate labour income) as the level parameter τ^N is moved within the $[0,1]$ interval. The orange curve shows total revenue from labour income taxation (i.e., $\mathcal{T}^N$ in equation (10), divided by initial output for scaling) as a function of the average effective labour tax rate. On this curve, the black dot represents the initial BGP, where labour tax revenue is 22.7% of GDP by construction (see Table 1). The red dot on the same curve shows the effective average labour tax rate and the associated tax revenue if the underlying tax parameter is adjusted until the new spending target is met (i.e., G/Y is 1.37 pp. higher than in the initial BGP). The orange dot is the top of this curve, i.e., the maximal tax revenue one can achieve by changing τ^N.

While the orange curve may suggest significant taxing capacity (about 7 pp. along the y-axis between the black and orange dots), this is misleading. As discussed above, and already illustrated in Table 4, changing one tax moves the entire equilibrium and thus alters *all revenue sources*. In the present case, increasing labour taxation indirectly contracts consumption tax revenues (due to lower consumption expenditures) and raises household transfers (since fewer households are active) as well as debt servicing (due to lower savings and associated increase in interest rates). Accordingly, the blue lines in Figure 9 show the level of public spending (also rescaled by initial GDP) consistent with the government budget constraint (equation (10)) when all adjustments in revenues and transfers are accounted for. At the initial BGP (the black dot), the government spends 24.1% of GDP, as reported in Table 1. While reaching the new spending target by adjusting τ^N remains feasible (the red dot on the blue line), this nearly exhausts the government's spending capacity (the blue dot on the blue line). This confirms the extent of the fiscal distortions associated with broad-based labour taxation.

Tax progressivity. As argued by Ferriere and Navarro (2025), raising the *progressivity* of the labour tax schedule may limit the adverse impact of labour taxation on aggregate labour supply, by specifically targeting high-earnings/low-LPE individuals. This is correct, as can readily be seen by looking at the aggregate employment response in Table 3, as well as their distribution across age and skill groups in Panels (c) of Figures 7 and 8.[14] In fact, employment increases rather than decreases, a consequence of the negative wealth effects on labour supply associated with rising government spending, which are now unmitigated by the strong substitution effects associated with broad-based taxation. As Figure 7 shows, the competition between wealth and substitution effects plays out differently at different ages. For the younger cohorts, who have relatively low labour earnings and are thus less affected by the tax change (but expect to be affected in the future as their earnings will rise), wealth effects dominate, leading to greater labour supply in the final BGP than in the initial BGP. In contrast, older cohorts have

[14] Note that raising ζ may increase notional transfers to households with very low incomes, a counterfactual property for a policy aimed at raising revenue. Following Ferriere and Navarro (2025), we preclude any such transfers as follows. Let ζ_0 denote the value of the progressivity parameter at the initial BGP, $\zeta_1 > \zeta_0$ its value at the final BGP, and $\tau(y;\zeta)$ the notional average tax rate for a household with gross earnings y under ζ. We assume that the *actual* average tax rate of this household is $\tau(y) = \max\{\tau(y;\zeta_0), \tau(y;\zeta_1)\}$, so that no one benefits from the increase of ξ from ξ_0 to ξ_1 (Of course, this correction requires a larger change in ξ to generate a given amount of revenue).

Figure 9: Laffer curves.



Notes: Each curve plots the set of BGPs obtained by continuously moving the tax parameter indicated in the panel's title. In Panels (a)-(b), the x-axes report the average effective labour tax rate (i.e., the ratio of aggregate labour tax revenues to aggregate labour income) implied by varying the level (τ^N) or progressivity (ζ) parameter of the labour tax schedule (4) between 0 and 1. In Panels (c)-(d), the x-axes report the tax wedges $\tau^K/(1+\tau^K)$ and $\tau^C/(1+\tau^C)$ (i.e., the shares of the pre-tax return to capital and consumption expenditures accruing to the government) implied by varying the tax instruments τ^K and τ^C between 0 and 1. In all panels, the orange curves plot the revenue from the specific tax under consideration, and the blue curves the implied affordable value of government spending given the endogenous adjustment of all tax bases (that is, considering G as the residual in the government budget constraint (10)). Both quantities are divided by initial output for scaling. The black dots correspond to the initial BGP (reported in Table 1), the red dots to the final BGP, and the orange and blue dots to the top of the corresponding curves.

higher asset wealth and shorter horizons, both of which contribute to weakening such wealth effects; this weakening of wealth effects with age ultimately turns labour-supply responses from positive to negative for the oldest cohorts.

While total employment does increase when the progressivity parameter is raised, targeting high-income households the way progressivity comes with two contrarian effects. First, it impedes savings, which are primarily driven by high-income households, and thus capital accumulation (K/Y in Table 3). Second, even though few households leave the labour force, those who do are also the most productive ones (whether due to ω or z), which is detrimental to TFP (see Table 3). Both effects ultimately generate substantial output and consumption losses when moving from the initial to the final

BGP, despite greater employment in the latter. Such detrimental effects of progressive taxation, which materialise mostly in the long run, are absent from Ferriere and Navarro (2025) due to their focus on short-lived spending shocks; our analysis indicates that the merits of progressive taxation for consumption crowding out that they highlight cannot be extrapolated to situations in which government spending increases durably.

One may again get insights into the macroeconomic and fiscal impacts of progressive taxation via the corresponding Laffer curves, as represented in Panel (b) of Figure 9. Note that changing the progressivity parameter within the admissible range implies smaller variations in the average effective labour tax rate (the x-axis) than with the level parameter. In any case, the maximal affordable amount of government spending (the top of the blue line), and the level of average effective labour tax rate at which it occurs, are comparable across the two parameters of the tax schedule. This shows that, while the exact tax distortions clearly differ across the two tax schedule parameters, they are substantial in both cases.

5.1.2 Other taxes

Consumption taxation. Among all four tax options under consideration in the upper half of Table 3, the consumption tax is that associated with the smallest contraction in consumption (−1.9%). The contrast with the macroeconomic impact of labour taxation may appear surprising. In the Representative Agent benchmark with an intensive margin of labour supply and proportional taxation, taxing labour or consumption is equivalent: equal revenues from either tax generate equal wealth effects, and those taxes are interchangeable in the labour wedge, i.e., the gap between the real wage and marginal rate of substitution between leisure and consumption. This equivalence breaks down under household heterogeneity and an extensive margin of labour supply, because labour taxation then falls entirely on working households (possibly deterring them from working altogether), while the burden of consumption taxation is broadly shared across the entire population. As a consequence, workers' labour wedge under consumption taxation is substantially lower, and so is workers' incentive to substitute away from consumption and towards leisure. Yet, the wealth effects associated with the greater present value of taxes remain, urging households as a whole to work *more* rather than less due to the higher present value of taxes. The impact on total employment is thus positive (+1.2% relative to the initial BGP according to Table 3). Because such wealth effects get weaker as the planning horizon shortens, the increase in employment is concentrated among the younger cohorts and declines with age (see Panel (e) of Figure 7). The proportional nature of the consumption tax also implies that wealth effects bite symmetrically across skill groups — higher skills means more gross income but also a greater present value of consumption taxes to be paid. However, since the medium-skilled group is larger than the others, so is its contribution to the overall increase in employment (see Panel (e) of figure 8).

Panel (d) of Figure 9 confirms how little distortionary consumption taxation is relative to labour taxation. The x-axis there displays the consumption tax wedge $\tau^C/(1+\tau^C) \in [0,1]$, i.e. the fraction of consumption expenditure devoted to consumption tax payments. The orange and blue lines respectively report the consumption tax revenue and affordable level of government spending (both rescaled by initial GDP, as usual). The widening of the consumption tax wedge necessary to finance the targeted increase in government spending over GDP is minimal and far from the maximum affordable level.

Capital taxation. Capital taxation lies somewhere between the labour taxes and the consumption tax in terms of output and aggregate consumption crowding out (Table 3). Unsurprisingly, the distortions it induces are primarily reflected in a low capital-output ratio (−4.4%): the tax discourages capital accumulation, ultimately reducing production (−1.6%). Here again, wealth effects on labour supply contribute to raising employment (+2.1%), thereby buffering the impact on output of the capital stock reduction. This occurs despite the fact that the reduction in capital stock lowers the marginal product of labour and, thereby, the demand for it at any given wage.

The Laffer curve for this tax (Panel (c) of Figure 9 suggests abundant fiscal capacity. Just as with consumption taxation, the x-axis here is the relevant tax wedge $\tau^K/(1+\tau^K) \in [0,1]$, i.e., the fraction of the gross return to capital accruing to the government, while the orange and blue line respectively depict the revenue from capital taxation and the affordable level of goverment spending in the final BGP (divided by initial GDP for scaling). Covering the fiscal shortfall requires a moderate increase in the capital tax wedge, and the maximum level of spending is much higher than the target (though not as high as under consumption taxation). Again, this is because wealth effects dominate the labour supply response to capital taxation, while substitution effects dominate the response to labour taxation.

5.1.3 Social security

Legal retirement age. Lastly, we consider the aggregate and distributional effects of financing the military buildup through Social Security rather than through tax changes. By construction of our benchmark path of government spending in Figure 6, its cost is exactly covered by a two-year shift in the legal retirement age (from 63 to 65) and no adjustment of any other fiscal parameter. Note that the *effective* (as opposed to *legal*) average shift is only 1.8 years. Recall that the legal retirement age is the age at which households become entitled to retirement benefits, not the age at which they would be pushed out of the labour force. Ultimately, an individual's decision to retire solves a trade-off (between collecting labour earnings versus a transfer), both terms of which depend on age since earnings are hump-shaped in age while transfers to non-working households jump up at $j = J^R$ (see Section 2.2). As a result of this trade-off, and given household heterogeneity, there is always a fraction of households that permanently switch to inactivity before having reached the legal retirement age, while another fraction keeps working for some years after reaching that age. In this context, all the shift in the legal age does is modify the terms of the tradeoff between staying at work and permanently exiting the labour force. The aggregation of those decisions yields a change in the average retirement age that need not match the shift in the legal age.

Bearing these considerations in mind, the shift in the legal retirement age generates enough revenue to cover the fiscal shortfall at virtually no cost in terms of consumption, whether aggregate (Table 3) or across the distribution of individuals (Panels (a) of Figures 7-8). Essentially, the buildup is fully paid for by a higher labour supply, eliminating the need to adjust consumption. While this increase is concentrated among directly affected households (i.e., those in the 60-70 age bracket of Figure 7), some other households respond too — namely, the 70+ (some of whom still work and further delay their exit from the labour force) and the 18-30 (who delay entry into the labour force, anticipating a longer period of activity over their lifetime).

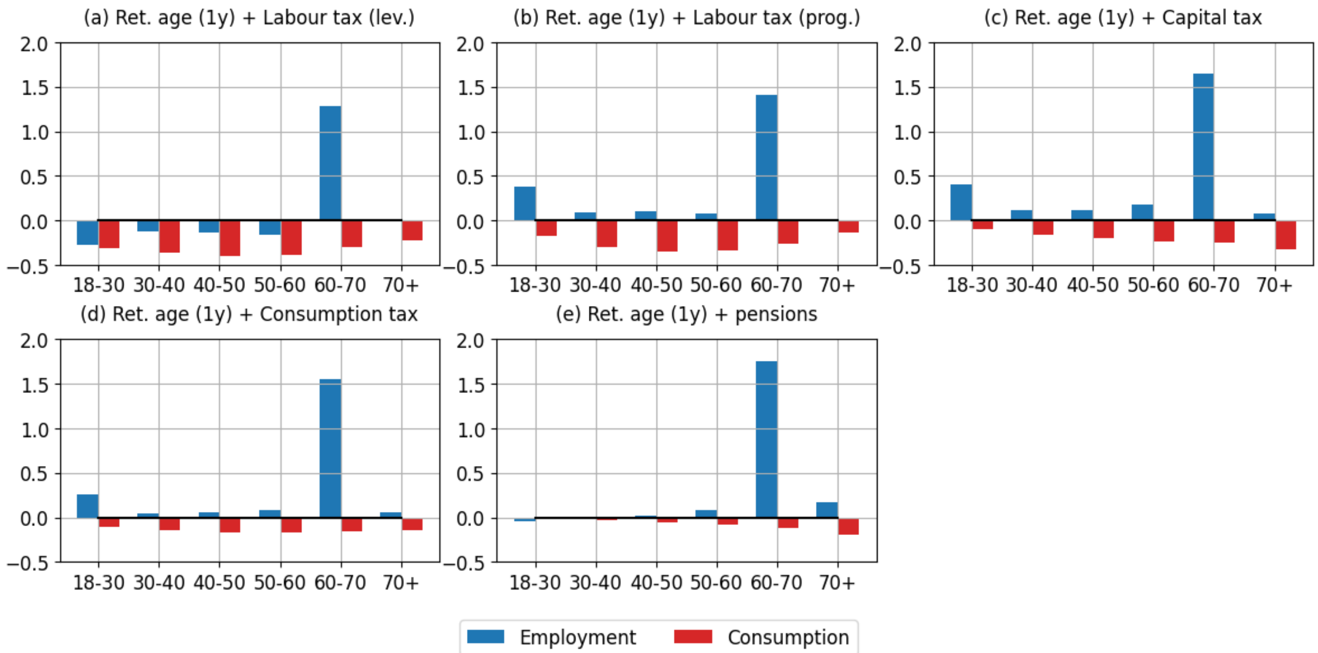
Retirement benefits. Another Social Security parameter that can be adjusted to address the fiscal shortfall is the generosity of retirement benefits. As shown in Table 3, the latter policy mitigates aggregate consumption crowding out relative to any form of taxation — though not as effectively as the shift in the legal retirement age. The driving forces underlying this moderate consumption response are the wealth effects on labour supply associated with the drop in the present value of pensions: aggregate consumption falls little (by 1.0%) because employment rises (by 1.1%). Since the cost of the pension cut in present-value terms increases as the time of the cut approaches, these wealth effects get stronger as individuals age, and so do their employment response, at least for those who are still sufficiently productive to work (Panel (f) of Figure 7). Such positive employment responses mitigate the need to cut consumption for middle-aged individuals, though cutting consumption is the only option for those aged 70+, since they are too unproductive for working to be a worthwhile option.

5.2 Mixed fiscal adjustments

Having dissected how each fiscal instrument affects the distribution of households and ultimately macro and fiscal aggregates, we may now turn to a subset of mixed adjustments whereby the burden of the buildup is spread over two fiscal instruments. For conciseness, we focus here on mixed adjustments all involving a one-year increase in the retirement age, complemented by a second fiscal instrument to cover the shortfall generated by the benchmark spending path in Figure 6. This choice is mainly for illustrative purposes and entails no loss of generality, as any combination of fiscal tools contemplated above can be implemented.

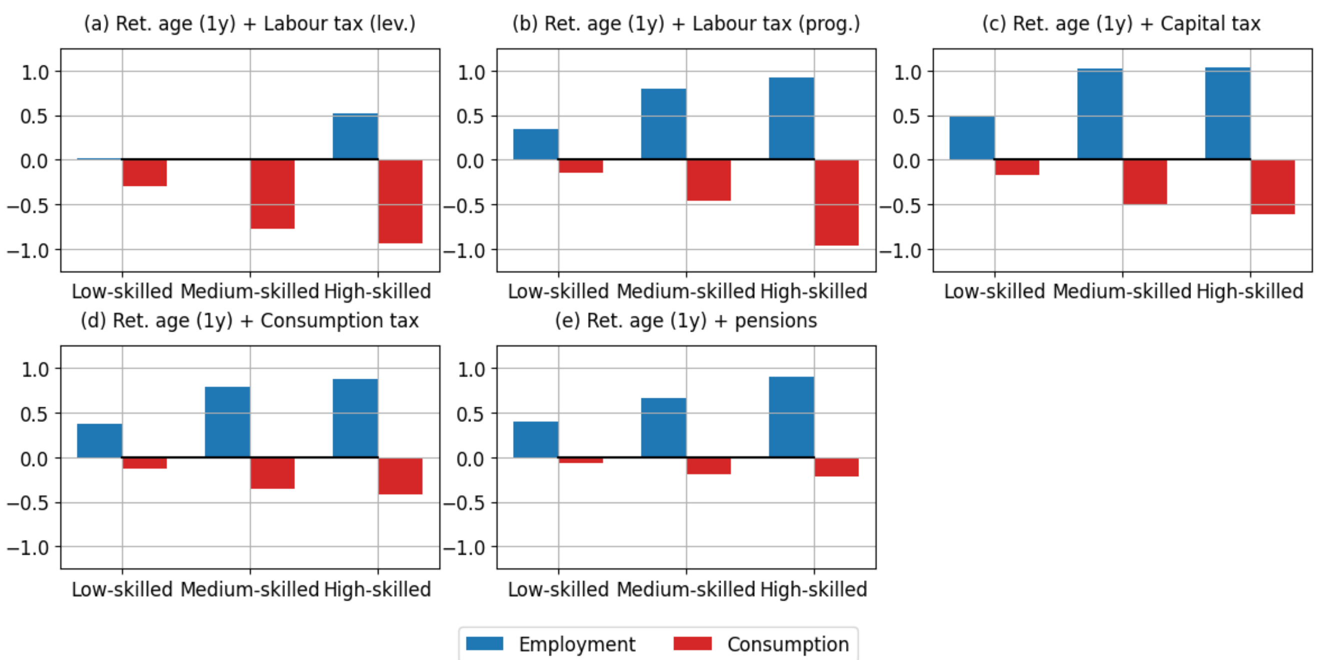
The macroeconomic and fiscal effects of mixed fiscal adjustments are gathered in the lower half of Tables 3 and 4, and their distributions across age and skill groups are shown in Figures 10 and 11. Mixed adjustments inherit the main properties of their single-instrument counterparts, with the one-year increase in the legal retirement age essentially acting as a scaled-down version of the two-year shift discussed above. Across all combinations, delayed retirement continues to expand employment at older ages and to reduce the need for additional taxation or pension cuts, so that the macroeconomic effects and fiscal revenue changes lie roughly between those of the pure retirement-age scenario and those of the second instrument in isolation. This is visible in the lower panel of 3, where output, employment and aggregate consumption responses for mixed reforms interpolate between

Figure 10: Contributions by age groups to the aggregate change (pp.) (mixed adjustments)



Notes: Each bar represents the contribution (in pp.) of a given group to the aggregate change in the variable (in % deviation from the initial BGP). This is equal to the % change in group-level consumption and employment *times* the fraction of the population belonging to that group.

Figure 11: Contributions by skill groups to the aggregate change (pp.) (mixed adjustments)



Notes: Each bar represents the contribution (in pp.) of a given group to the aggregate change in the variable (in % deviation from the initial BGP). This is equal to the % change in group-level consumption and employment *times* the fraction of the population belonging to that group.

the corresponding single-instrument entries, and in 4, where the adjustment of labour taxes, capital taxes, consumption taxes or pensions contributes less to the overall revenue increase than under single-handed reforms. In other words, mixed reforms smooth the fiscal burden across instruments but do not qualitatively alter the ranking established in Section 5.1: combinations with labour-tax hikes still entail relatively strong consumption crowding-out, those involving capital taxation continue to depress the capital–output ratio, while packages relying on consumption taxes or pension cuts remain comparatively benign in terms of aggregate losses.

The distributional patterns across age and skill groups further confirm that mixed packages behave almost like convex combinations of their building blocks. Figures 10 and 11 show that the extra participation induced by postponing retirement is now more modest and concentrated in the 60–70 age bracket, while the second instrument shapes who ultimately bears the residual adjustment: labour-tax mixes shift some of the burden towards prime-age and medium-skilled workers, capital-tax mixes tilt it towards high-skilled wealth holders, and pension-based mixes concentrate the incidence on current and future retirees. Relative to their single-instrument counterparts, these mixed reforms therefore dampen the extreme responses associated with any one tool, but they do not generate qualitatively new margins of adjustment nor a markedly better trade-off between aggregate consumption crowding-out and distributional impacts; they mainly offer a more "averaged" version of the pure scenarios previously analysed.

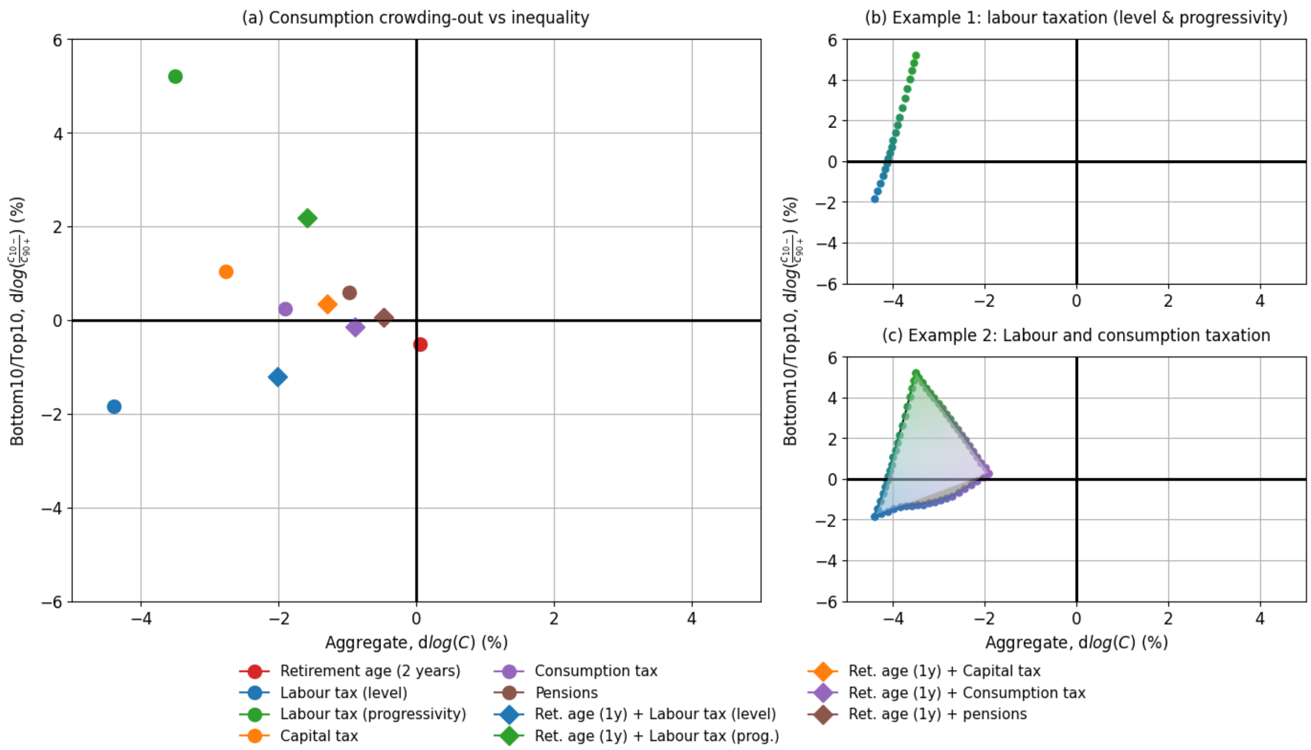
6 Aggregate consumption crowding-out versus inequality

We conclude our analysis by evaluating how different fiscal adjustments affect the distribution of consumption. Our focus on consumption is motivated by the observation that, in Representative-Agent analyses, a traditional descriptive measure of the private cost of public spending is the amount of crowding out of private consumption it generates. In our Heterogeneous-Agent model, this would correspond to the percentage decline in aggregate consumption when moving from the initial to the final BGP (the last column of Table 3), though this calculation ignores the distributional effects at play. A natural question to ask is how the various fiscal adjustments under consideration compare in terms of consumption *inequality*, and how this weighs against the reduction in aggregate consumption.

To address this question, Figure 12 plots the percentage fall in aggregate consumption when moving from the initial to the final BGP (along the x-axis) against the changes in inter-decile consumption ratio (along the y-axis). The latter measures how much the bottom 10% of the population (ranked by consumption) catches up to or falls further behind the top 10% after the policy change. Both single-handed and mixed adjustments are considered, and are respectively represented by dots and diamonds in the Figure. The locations of the various points in the scatterplot inform us about how the different fiscal adjustments considered before perform across these two metrics.

The first striking feature of this chart is that adjusting the level parameter of the labour tax schedule, whether in isolation (blue dot) or combined with a one-year shift in the legal retirement age (blue diamond), performs unequivocally badly. Secondly, some other fiscal adjustments, such as a pure

Figure 12: Aggregate consumption crowding out versus changes in consumption inequality.



Notes: The three panels plots the % fall in aggregate consumption when moving from the initial to the final BGP (x-axis) against the changes in inter-decile consumption ratio (y-axis). Households are ranked by consumption levels, and the ratio reports the average consumption of the bottom 10% over that of the top 10%. Panel (a) plots the outcomes of all pure and mixed fiscal adjustments considered in Sections 5.1-5.2. Panel (b) plots the set of outcomes that can be achieved by increasing the level and progressivity of the labour tax schedule while matching the benchmark spending target. Panel (c) extends the set of Panel (b) to any combinations of labour and consumption taxation.

capital tax hike or a pure consumption tax hike, perform relatively better than fiscal adjustments involving broad-based labour taxation, yet display relatively poor performance in the sense of combining significant aggregate consumption losses with a very small reduction in inequality. Lastly, the fiscal adjustments that remain in the race reveal a tradeoff between aggregate consumption crowding out and consumption inequality: the fiscal authority cannot hope to mitigate the fall in the former while at the same time reducing the latter. A noticeable case in point is a fiscal adjustment through pure labour tax progressivity, which does best in terms of inequality reduction while doing almost worst in terms of aggregate consumption crowding out.

Could some combinations of fiscal adjustments other than those considered in Section 5.2 strike a better balance between the two considerations? To answer this question, consider any set of single-handed fiscal or social security instruments and the associated set of dots in Figure 12. Any combination of those instruments must lie in some area between those dots. For example, any combination of the level and progressivity parameters yielding the targeted amount of government spending must lie between the blue and green dots, as depicted on Panel (b) of Figure 12. If the mix of instruments also includes the consumption tax (say), then any mix of the three instruments must lie within the area between the green, blue, and purple dots, which is roughly triangular with those dots as vertices, as shown on Panel (c) Eventually, visual inspection of Figure 12 shows that the policies yielding the best tradeoff between aggregate consumption and consumption inequality must combine the following instruments: a *shift in the legal retirement age* (the red dot, for the single-handed version), a *cut in pensions* (the brown dot), and an *increase in the progressivity parameter* of labour tax schedule (the green dot). Of course, the performance of a given set of fiscal instruments along these two dimensions (aggregate consumption loss versus inequality) is conditional on the extent of pre-existing tax distortions at the initial BGP. For example, the labour tax wedge is already substantial in France, so it may not be surprising that further widening it through broad-based taxation performs poorly. Calibrating our framework to a different country may accordingly exclude or favour a different set of fiscal instruments.

7 Conclusion

This paper has quantified how a permanent increase in government spending of the size implied by the new NATO "core defence" target affects aggregates and distributions, depending on how it is financed. The analysis uses a calibrated OLG-HA model with life-cycle heterogeneity, earnings risk, and a detailed fiscal system, and tracks how financing choices change labour supply, saving, and ultimately output, capital, and consumption across groups. This has led us to revisit the study of large, persistent changes in government spending, which has hitherto been mostly confined to the Representative-Agent framework.

According to our model, calibrated to the French economy, increasing the legal retirement age by two years is the only single-instrument reform that can support the target spending path with almost no long-run reduction in aggregate private consumption; higher defence spending is essentially paid for by more labour market participation, especially at older ages. Cutting pensions also raises labour supply and limits aggregate crowding-out, but reduces consumption for current and future retirees. By contrast, financing the same spending path with higher labour or capital taxes leads to sizeable long-run falls in output and consumption: broad-based labour tax hikes shrink employment and the tax base, progressive labour tax hikes reduce saving and capital, and higher capital taxes depress investment. A higher consumption tax produces the smallest aggregate losses among the tax instruments, because it spreads the burden across workers and non-workers and induces relatively small employment distortions.

Combining a one-year increase in the retirement age with tax or pension changes spreads the adjustment across instruments but does not remove the basic trade-off between aggregate consumption losses and consumption inequality. When we map all scenarios in this two-dimensional space, no financing scheme delivers both very small aggregate crowding-out and lower consumption inequality. The set of relatively favourable options all involve some use of social-security parameters (retirement age and/or pension levels), possibly combined with limited changes in labour-tax progressivity or consumption taxation, while heavy reliance on any single tax instrument performs poorly on at least one of the two dimensions.

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Appendix

A Model Environment

A.1 Demographics

Let $L_{18,t}$ denote an exogenous, stationary inflow of newcomers. For ages $j \in \boldsymbol{J} \equiv \{18,\dots,J\}$, the cohort size $L_{j,t}$ evolves according to

$$L_{j+1,t+1} = \psi_{j+1}\big(L_{j,t} + M_{j,t}\big) = L_{j,t}\psi_{j+1}(1+m_j), \tag{A.1}$$

where $\psi_{j+1} \in [0,1]$ is the survival rate from age j to $j+1$ and $m_j \equiv M_{j,t}/L_{j,t}$ is an exogenous, stationary inflow of migrants of age j arriving at the end of period t—before mortality is realised—so they neither consume nor supply labour within period t. Newborns arrive at a constant rate n, so that $L_{1,t+1} = (1+n)L_{1,t}$, with $L_{1,0}$ normalised to one. We define total population $L_t \equiv \sum_{j\in\boldsymbol{J}} L_{j,t}$,[15] whose growth rate is also n under stationarity.[16] Let the cohort share be $\mu_j \equiv L_{j,t}/L_t$, which is stationary by construction, and note that it follows

$$\mu_{j+1} = \frac{\mu_j \psi_{j+1}(1+m_j)}{1+n}. \tag{A.2}$$

A.2 The Household Problem

A.2.1 Problem Statement

The state of an individual household is defined by its skill/education group $\omega > 0$, with $\omega \in \boldsymbol{\Omega} \equiv \{\omega_1,\dots,\omega_{N_\omega}\}$, which can equivalently be interpreted as its permanent productivity type. The stationary distribution of these groups is denoted by $\pi^{\Omega}(\omega)$. Each household is also characterised by its age $j \in \boldsymbol{J}$ and by an idiosyncratic, transitory productivity shock $z > 0$, where $z \in \boldsymbol{Z} \equiv \{z_1,\dots,z_{N_z}\}$, governed by the transition matrix Π^Z and stationary distribution $\pi^Z(z)$. Let o denote the household's occupational status, $o \in \boldsymbol{O} = \{E, I\}$, corresponding to either employed (E) or inactive (I).

In each period t, the household's state also includes its wealth, comprising bond holdings $b \in \boldsymbol{B}_{t-1} \equiv \left\{b = b\Gamma_{t-1} : b \in \boldsymbol{B} \equiv [0,\overline{b}]\right\}$ with $0 < \overline{b}$, and corporate claims $k \in \boldsymbol{K}_{t-1} \equiv \left\{k = k\Gamma_{t-1} : k \in \boldsymbol{K} \equiv [0,\overline{k}]\right\}$ with $0 < \overline{k}$. The term Γ_t denotes an exogenous productivity trend evolving according to $\Gamma_t = (1+\gamma)\Gamma_{t-1}$, where $\gamma > 0$ and initial level $\Gamma_{-1} > 0$ are given.The stationary growth rate of the economy is thus defined as $g \equiv (1+\gamma)(1+n)$.

[15] L_t is the total population at the beginning of period t.

[16] For all $j \geq 19$ and $t \geq J-1$, $L_{j,t} = L_{18,t}\prod_{\ell=18}^{j-1}\frac{\psi_{\ell+1}(1+m_\ell)}{1+n}$, hence each $L_{j,t}$ is proportional to $L_{18,t}$ and so is L_t.

Let $\mathscr{H}_t(\omega, j, z, b, k; o)$ denote the correspondence

$$\begin{aligned}\mathscr{H}_t(\omega, j, z, b, k; o) = \Big\{(c, b', k') :& \\ (1+\tau_t^C)c + b' + k' = (1 + r_{b,t})\Big(b + \xi_t^b(\omega, j)\Big) &+ (1 + r_{k,t})\Big(k + \xi_t^k(\omega, j)\Big) \\ &+ \mathrm{tr}_t(\omega, j, o) + \mathbb{1}[o \in \{E\}]\left(\bar{h}_j e_j \omega z w_t - \mathscr{T}_t(\bar{h}_j e_j \omega z w_t)\right); \\ &b' \geq 0; k' \geq 0; c \geq 0\Big\},\end{aligned} \tag{A.3}$$

which represents the individual's feasible choices of consumption, c, and savings in the form of bonds, b', and claims, k', conditional on whether the individual decides to participate in the labour market, $o = E$, or not, $o = I$. Consumption is subject to a consumption tax τ_t^C. The returns of bonds and claims are $r_{b,t}$ and $r_{k,t}$ respectively. A fraction of these assets generates state-dependent bequests from one period to the next; these bequests earn interest and are distributed in period t: they are denoted accordingly by $\xi_t^b(\omega, j)$ and $\xi_t^k(\omega, j)$, respectively. Households receive state-contingent transfers tr_t. An agent belonging to cohort j who participates to the labour market, earns gross labour income $\bar{h}_j e_j \omega z w_t$ where hours worked, $\bar{h}_j$, are age-dependent and exogenously supplied. w_t is the wage rate and $e_j \omega z$ is the individual productivity. They pay labour tax $\mathscr{T}_t(\cdot)$ so that post-tax earnings are:

$$\bar{h}_j e_j \omega z w_t - \mathscr{T}_t(\bar{h}_j e_j \omega z w_t) = (1 - \tau_t^N)\left(\bar{h}_j e_j \omega z w_t\right)^{1-\zeta_t} \Gamma_t^{\zeta_t}$$

where τ_t^N and ζ_t represent the labour income tax level and progressivity parameters, respectively. The term $\Gamma_t^{\zeta_t}$ ensures that, despite progressivity, the after-tax labour income grows at the same rate as consumption or assets. Finally, $\mathbb{1}[\mathscr{P}]$ defines an indicator function which returns one if proposition $\mathscr{P}$ is true, zero otherwise; we also define the function $\mathbb{1}_E(x) \equiv \mathbb{1}[x \in E]$ for any set E.

Let $V_t(\omega, j, z, b, k, o_-)$ denote the value of an individual in state $(\omega, j, z, b, k, o_-)$, where o_- represents the occupational status before labour market participation choices. It is worthwhile to mention that for cohorts that have not yet reached retirement age (i.e., $j < J^R < J$), this additional state is not relevant, but since retirement is an absorbing state, keeping this information would help to keep track of retirees.

The individual problem reads

$$V_t(\omega, j, z, b, k, o_-) = \mathbb{E}_{\chi_t(E), \chi_t(I)}\left[\max_{o \in \{E, I\}}\left\{W_t(\omega, j, z, b, k; o) + \chi_t(o)\Gamma_t^{1-\sigma}\right\}\right], \tag{A.4}$$

where

$$W_t(\omega, j, z, b, k; o) = \max_{(c,b',k') \in \mathcal{H}_t(\omega,j,z,b,k;o)} \left\{ \frac{\left(c + \frac{\alpha b'}{1+\tau_t^C}\right)^{1-\sigma}}{1-\sigma} - \mathbb{1}[o \in \{E\}]\kappa_{\omega,j}\Gamma_t^{1-\sigma} \right.$$
$$\left. + (1-\psi_{j+1})\nu_a\Gamma_t^{\varphi-\sigma}\frac{\left(\frac{k'+(1-\alpha)b'}{1+\tau_t^C}\right)^{1-\varphi}}{1-\varphi} + \beta\psi_{j+1}\sum_{z'\in\boldsymbol{Z}}\Pi^Z(z,z')V_{t+1}(\omega, j+1, z', b', k', o) \right\}, \quad \text{(A.5)}$$

with $\beta \in (0,1)$ the subjective discount factor. The presence of assets in the utility function reflects both a bequest motive, parameterised by a scale factor $\nu_a > 0$ and a curvature parameter $\varphi \le \sigma$ that governs the extent to which bequests are a luxury good, and a liquidity motive, parameterised by $\alpha \in [0,1]$. The parameter $\kappa_{\omega,j}$ governs the age- and skill-specific labour dis-utility. $\chi_t(o)$ is a zero-mean type-I Extreme-Value taste shock affecting the discrete labour participation choice, and $\mathbb{E}_{\chi_t(E),\chi_t(I)}$ is the expectation operator with respect to the two taste shocks.

For cohorts above potential retirement age (i.e., $j \ge J^R$), the occupational status $o = I$ is absorbing and is interpreted as retirement. As a consequence, we need an extra individual state to keep track of retirees. Let o_- denote the individual occupational state before labour market participation choices. The individual problem becomes

$$V_t(\omega, j, z, b, k, o_- = E) = \mathbb{E}_{\chi_t(E),\chi_t(I)}\left[\max_{o\in\{E,I\}}\left\{W_t(\omega, j, z, b, k; o) + \chi_t(o)\Gamma_t^{1-\sigma}\right\}\right], \quad \text{(A.6)}$$

and

$$V_t(\omega, j, z, b, k, o_- = I) = W_t^R(\omega, j, b, k),\ \forall z \in \boldsymbol{Z}, \quad \text{(A.7)}$$

with

$$W_t(\omega, j, z, b, k; o = E) = \max_{(c,b',k') \in \mathcal{H}_t(\omega,j,z,b,k;E)} \left\{ \frac{\left(c + \frac{\alpha b'}{1+\tau_t^C}\right)^{1-\sigma}}{1-\sigma} - \kappa_{\omega,j}\Gamma_t^{1-\sigma} \right.$$
$$\left. + (1-\psi_{j+1})\nu_a\Gamma_t^{\varphi-\sigma}\frac{\left(\frac{k'+(1-\alpha)b'}{1+\tau_t^C}\right)^{1-\varphi}}{1-\varphi} + \beta\psi_{j+1}\sum_{z'\in\boldsymbol{Z}}\Pi^Z(z,z')V_{t+1}(\omega, j+1, z', b', k', o = E) \right\}, \quad \text{(A.8)}$$

and $W_t(\omega, j, z, b, k; o = I) = W_t^R(\omega, j, b, k)$ for all $z \in \boldsymbol{Z}$, with

$$W_t^R(\omega, j, b, k) = \max_{(c,b',k') \in \mathscr{H}_t(\omega,j,z,b,k;I)} \left\{ \frac{\left(c + \frac{\alpha b'}{1+\tau_t^C}\right)^{1-\sigma}}{1-\sigma} + (1-\psi_{j+1}) \nu_a \Gamma_t^{\varphi-\sigma} \frac{\left(\frac{k'+(1-\alpha)b'}{1+\tau_t^C}\right)^{1-\varphi}}{1-\varphi} + \beta \psi_{j+1} W_{t+1}^R(\omega, j+1, b', k') \right\}. \quad \text{(A.9)}$$

A.2.2 Portfolio Problem and Arbitrage Condition

The problem (A.5) leads to the following first order conditions with respect to k' and b'

$$-\frac{1}{1+\tau_t^C}\left(c + \frac{\alpha b'}{1+\tau_t^C}\right)^{-\sigma} + \frac{1-\psi_{j+1}}{1+\tau_t^C} \nu_a \Gamma_t^{\varphi-\sigma} \left(\frac{k' + (1-\alpha) b'}{1+\tau_t^C}\right)^{-\varphi} + \beta \psi_{j+1} \sum_{z' \in \boldsymbol{Z}} \Pi^Z(z, z') V_{k,t+1} + \eta^k = 0, \quad \text{(A.10)}$$

$$-\frac{1-\alpha}{1+\tau_t^C}\left(c + \frac{\alpha b'}{1+\tau_t^C}\right)^{-\sigma} + (1-\alpha)\frac{1-\psi_{j+1}}{1+\tau_t^C} \nu_a \Gamma_t^{\varphi-\sigma} \left(\frac{k' + (1-\alpha) b'}{1+\tau_t^C}\right)^{-\varphi} + \beta \psi_{j+1} \sum_{z' \in \boldsymbol{Z}} \Pi^Z(z, z') V_{b,t+1} + \eta^b = 0, \quad \text{(A.11)}$$

where η^k and η^b are the two Kuhn-Tucker multipliers associated with the positivity constraints on k' and b', respectively.[17] Combining these conditions for an interior solution (i.e., $\eta^b = \eta^k = 0$) and the envelope conditions

$$V_{k,t} = \frac{1+r_{k,t}}{1+\tau_t^C}\left(c + \frac{\alpha b'}{1+\tau_t^C}\right)^{-\sigma} \quad \text{and} \quad V_{b,t} = \frac{1+r_{b,t}}{1+\tau_t^C}\left(c + \frac{\alpha b'}{1+\tau_t^C}\right)^{-\sigma}, \quad \text{(A.12)}$$

leads to the sufficient condition between the rate of returns on claims and bonds

$$(1-\alpha)(1+r_{k,t+1}) = 1 + r_{b,t+1}, \quad \text{(A.13)}$$

which leads to an equivalence between (A.10) and (A.11). Note also that if (A.13) holds and the individual chooses, say, $k' = 0$, without loss of generality, then, it must be the case that $\eta^b, \eta^k > 0$, and hence, $b' = 0$ is also optimal. Finally, since optimisation problems (A.8) and (A.9) share the same structure as problem (A.5), we obtain the same conclusions for the case where $j \geq J^R$, regardless of whether the individual chooses to remain employed or to become inactive (i.e., retired).

[17]Note that due to the CRRA form of the utility function w.r.t. c, we know that choosing $c \leq 0$ is suboptimal.

All this means that given the equilibrium condition (A.13), if the Euler condition for capital claims holds, then the one for bonds also holds; the latter is not an additional equation. Moreover, only one rate of return is relevant to household savings choices; the other one is a translated—hence not independent—version of that return. Finally, *all* households are exactly indifferent between holding capital claims and government bonds (because the extra utility provided by the bonds is exactly offset by the lower returns). As a consequence, portfolios are indeterminate and must be assumed. Therefore, we assume them to be symmetric across age groups and to correspond to aggregate proportions. As households are indifferent between the nature of assets in equilibrium, we define assets $a \equiv b + k$, $a \in \boldsymbol{A}_{t-1} \equiv \left\{a = a\Gamma_{t-1} : a \in \boldsymbol{A} \equiv [0, \overline{a}]\right\}$, with $\overline{a} > 0$, so that at date t

$$k = \frac{K_{t-1}}{S_{t-1}} a \quad \text{and} \quad b = \frac{B_{t-1}}{S_{t-1}} a, \tag{A.14}$$

where S_{t-1} is the aggregate savings supplied at the end of period $t-1$, K_{t-1} the aggregate capital stock and B_{t-1} the aggregate stock of public debt (i.e., in equilibrium, $S_{t-1} = K_{t-1} + B_{t-1}$). Note that by the same line of reasoning, we can define state-contingent bequests $\xi_t(\omega, j)$ so that

$$\xi_t^k(\omega, j) = \frac{K_{t-1}}{S_{t-1}} \xi_t(\omega, j) \quad \text{and} \quad \xi_t^b(\omega, j) = \frac{B_{t-1}}{S_{t-1}} \xi_t(\omega, j). \tag{A.15}$$

Let Υ_t define the share of aggregate savings that do yield liquidity preference, that is

$$0 \leq \Upsilon_t \equiv \frac{\alpha B_t}{S_t} \leq 1. \tag{A.16}$$

Alternatively, $0 \leq 1 - \Upsilon_t \leq 1$ also represents a tilt on the return on capital claims to generate the return on assets. Indeed, let r_t denote the interest rate yielded by assets a, that is,

$$(1 + r_t) a = (1 + r_{b,t}) b + (1 + r_{k,t}) k. \tag{A.17}$$

We then have

$$1 + r_t = (1 + r_{k,t})(1 - \Upsilon_{t-1}). \tag{A.18}$$

A.2.3 A Reformulation of the Individual Problem

Since the variables b and k play the same role as a, with a slight abuse of notation, we may rewrite the individual problem as follows, starting with the correspondence

$$\begin{aligned}\mathscr{H}_t(\omega, j, z, a; o) = \Big\{&(c, a') : (1 + \tau_t^C) c + a' = (1 + r_t)\left(a + \xi_t(\omega, j)\right) \\ &+ \mathrm{tr}_t(\omega, j, o) + \mathbb{1}[o \in \{E\}]\left(\bar{h}_j e_j \omega z w_t - \mathscr{T}_t(\bar{h}_j e_j \omega z w_t)\right); a' \geq 0; c \geq 0\Big\}.\end{aligned} \tag{A.19}$$

Then, the individual problem can be rewritten for $j < J^R$

$$V_t(\omega, j, z, a, o_-) = \mathbb{E}_{\chi_t(E),\chi_t(I)}\left[\max_{o\in\{E,I\}}\left\{W_t(\omega, j, z, a; o) + \chi_t(o)\Gamma_t^{1-\sigma}\right\}\right], \tag{A.20}$$

where

$$W_t(\omega, j, z, a; o) = \max_{(c,a')\in\mathscr{H}_t(\omega,j,z,a;o)}\left\{\frac{\left(c + \frac{\Upsilon_t}{1+\tau_t^C}a'\right)^{1-\sigma}}{1-\sigma} - \mathbb{1}[o\in\{E\}]\kappa_{\omega,j}\Gamma_t^{1-\sigma}\right.$$
$$\left. + (1-\psi_{j+1})\nu_a\Gamma_t^{\varphi-\sigma}\frac{\left(\frac{1-\Upsilon_t}{1+\tau_t^C}a'\right)^{1-\varphi}}{1-\varphi} + \beta\psi_{j+1}\sum_{z'\in\boldsymbol{Z}}\Pi^Z(z,z')V_{t+1}(\omega, j+1, z', a', o)\right\}, \tag{A.21}$$

and for $j \geq J^R$, it reads, for a previously active individual

$$V_t(\omega, j, z, a, o_- = E) = \mathbb{E}_{\chi_t(E),\chi_t(I)}\left[\max_{o\in\{E,I\}}\left\{W_t(\omega, j, z, a; o) + \chi_t(o)\Gamma_t^{1-\sigma}\right\}\right], \tag{A.22}$$

with

$$W_t(\omega, j, z, a; o = E) = \max_{(c,a')\in\mathscr{H}_t(\omega,j,z,a;E)}\left\{\frac{\left(c + \frac{\Upsilon_t}{1+\tau_t^C}a'\right)^{1-\sigma}}{1-\sigma} - \kappa_{\omega,j}\Gamma_t^{1-\sigma}\right.$$
$$\left. + (1-\psi_{j+1})\nu_a\Gamma_t^{\varphi-\sigma}\frac{\left(\frac{1-\Upsilon_t}{1+\tau_t^C}a'\right)^{1-\varphi}}{1-\varphi} + \beta\psi_{j+1}\sum_{z'\in\boldsymbol{Z}}\Pi^Z(z,z')V_{t+1}(\omega, j+1, z', a', o = E)\right\}, \tag{A.23}$$

and

$$W_t(\omega, j, z, a; o = I) = W_t^R(\omega, j, a), \text{ for all } z \in \boldsymbol{Z}, \tag{A.24}$$

and for previously inactive individuals (retirees)

$$V_t(\omega, j, z, a, o_- = I) = W_t^R(\omega, j, a),\ \forall z \in \boldsymbol{Z}, \tag{A.25}$$

with

$$W_t^R(\omega,j,a)=\max_{(c,a')\in\mathcal{H}_t(\omega,j,z,a;I)}\left\{\frac{\left(c+\frac{\Upsilon_t}{1+\tau_t^C}a'\right)^{1-\sigma}}{1-\sigma}\right.$$
$$\left.+(1-\psi_{j+1})\nu_a\Gamma_t^{\varphi-\sigma}\frac{\left(\frac{1-\Upsilon_t}{1+\tau_t^C}a'\right)^{1-\varphi}}{1-\varphi}+\beta\psi_{j+1}W_{t+1}^R(\omega,j+1,a')\right\}. \quad \text{(A.26)}$$

Note that a' must belong to $\boldsymbol{A}_t$ in equations (A.19) to (A.26).

Accordingly, we have performed the following substitutions $V_t(\omega,j,z,b,k,o_-)\leftrightarrow V_t(\omega,j,z,a=b+k,o_-)$ which denotes the value of an individual in state $s=(\omega,j,z,a,o_-)\in\boldsymbol{S}_{t-1}\equiv\boldsymbol{\Omega}\times\boldsymbol{J}\times\boldsymbol{Z}\times\boldsymbol{A}_{t-1}\times\boldsymbol{O}$. Similarly, $W_t(\omega,j,z,b,k;o)\leftrightarrow W_t(\omega,j,z,a;o)$ and $W_t^R(\omega,j,b,k)\leftrightarrow W_t^R(\omega,j,a)$.

We conjecture that the conditions for the value functions to be homogeneous of degree $1-\sigma$ in assets are satisfied, with $\sigma>1$ the relative risk aversion coefficient (see Jones et al., 2005). For later reference, given the individual state $s\in\boldsymbol{S}_{t-1}$, we let $\hat{V}_t:\boldsymbol{S}\to\mathbb{R}$, with $\boldsymbol{S}\equiv\boldsymbol{\Omega}\times\boldsymbol{J}\times\boldsymbol{Z}\times\boldsymbol{A}\times\boldsymbol{O}$, denote the (stationary) value function so that $V_t(\omega,j,z,a,o_-)=\hat{V}_t(\omega,j,z,a,o_-)\Gamma_t^{1-\sigma}$.[18] We also define $\hat{W}_t(\cdot;o):\boldsymbol{\Omega}\times\boldsymbol{J}\times\boldsymbol{Z}\times\boldsymbol{A}\to\mathbb{R}$, for each $o\in\boldsymbol{O}$, and $\hat{W}_t^R:\boldsymbol{\Omega}\times\boldsymbol{J}\times\boldsymbol{A}\to\mathbb{R}$ so that $W_t(\omega,j,z,a;o)=\hat{W}_t(\omega,j,z,a;o)\Gamma_t^{1-\sigma}$ and $W_t^R(\omega,j,a)=\hat{W}_t(\omega,j,a)\Gamma_t^{1-\sigma}$.

Similarly, for a given occupation decision $o\in\boldsymbol{O}$ and the implied choice-specific solutions a' and c to the optimization problems (A.21), (A.23), and (A.26) in period t, we define the function $a'_t(\cdot;o):\boldsymbol{S}_{t-1}\ni s\mapsto a'\in\boldsymbol{A}_t$ and we let $g_{a,t}(\cdot;o):\boldsymbol{S}\to\boldsymbol{A}$, denote the detrended, choice-specific decision rule on assets, such that, $a'_t(\cdot;o)=g_{a,t}(\cdot;o)\Gamma_t$. Similarly, we define the function $c_t(\cdot;o):\boldsymbol{S}_{t-1}\ni s\mapsto c\in\mathbb{R}^+$ and we let $g_{c,t}(\cdot;o):\boldsymbol{S}\to\mathbb{R}^+$ denote the (also stationary and choice-specific) decision rule on consumption so that $c_t(\cdot;o)=g_{c,t}(\cdot;o)\Gamma_t$.

Finally, let us define $p_t(o\mid s)$ the probability of choosing o given an initial individual state $s=(\omega,j,z,a,o_-)$ if given the opportunity of choosing one's occupational status. We have

$$p_t(o=E\mid\omega,j,z,a,o_-)=\int_{\mathbb{R}}\int_{\mathbb{R}}\mathbb{1}\left[W_t(\omega,j,z,a;o=E)+\chi(E)\Gamma_t^{1-\sigma}\geq W_t(\omega,j,z,a;o=I)+\chi(I)\Gamma_t^{1-\sigma}\right]$$
$$\times f_\chi\left(\chi(E)\right)f_\chi\left(\chi(I)\right)d\chi(E)d\chi(I), \quad \text{(A.27)}$$

and $p_t(o=I\mid\omega,j,z,a,o_-)=1-p_t(o=E\mid\omega,j,z,a,o_-)$.

Armed with this, let us now define $P_t(o\mid s)$ the probability of choosing o given an initial individual state $s=(\omega,j,z,a,o_-)$:

$$P_t(o\mid s)=\begin{cases}p_t(o\mid s) & \text{if}\quad (j<J^R)\vee((j\geq J^R)\wedge(o_-=E)),\\ \mathbb{1}[o\in\{I\}] & \text{otherwise.}\end{cases} \quad \text{(A.28)}$$

[18]Note that due to the homogeneity of degree $1-\sigma$ in assets, this is equivalent to have $V_t(\omega,j,z,a,o_-)=V_t(\omega,j,z,a/(1+\gamma),o_-)\Gamma_t^{1-\sigma}$ for $a\in\boldsymbol{A}$, and then $\hat{V}_t(\omega,j,z,a,o_-)\equiv V_t(\omega,j,z,a/(1+\gamma),o_-)$.

A.3 Production

The final good is produced by a representative firm, according to

$$Y_t = Z\big(K_{t-1}\big)^{\theta}\big(\Gamma_t N_t\big)^{1-\theta}, \tag{A.29}$$

with $Z > 0$ a scale factor (total factor productivity), and $K_{-1} > 0$ given. The representative firm rents capital and labour on the economy-wide markets. Given a depreciation rate of δ and investment I_t, capital accumulation follows a standard law of motion

$$K_t = (1-\delta)K_{t-1} + I_t. \tag{A.30}$$

The cost of capital is $(1+\tau_t^K)r_{k,t} + \delta$ where τ_t^K is a corporate income tax. The associated first-order conditions are

$$(1+\tau_t^K)r_{k,t} + \delta = \theta Z\left(\frac{K_{t-1}}{\Gamma_t N_t}\right)^{\theta-1},$$

$$w_t = (1-\theta)Z\Gamma_t\left(\frac{K_{t-1}}{\Gamma_t N_t}\right)^{\theta}.$$

A.4 Transfers and Bequests

Let $\lambda_t(\omega, j, z, a, o_-)$ denote the distribution of individual state $s = (\omega, j, z, a, o_-) \in \boldsymbol{S}_{t-1}$ at the beginning of period t, immediately after having drawn individual productivity z. Let assume λ_0 be given and for any set $\boldsymbol{E}$, let us define $\mathscr{B}(\boldsymbol{E})$ the corresponding Borel subsets.

A.4.1 Transfers

State-contingent transfers include minimum income in case of inactivity and retirement pensions. These transfers are therefore calculated based on age and occupational status as follows:

$$\mathrm{tr}_t(\omega, j, o) \equiv w_t\phi_t(\omega, j, o) + T_t, \tag{A.31}$$

with T_t a lump-sum transfer and

$$\phi_t(\omega, j, o) = \begin{cases} 0 & \text{if} \quad (o = E), \\ \phi^I & \text{if} \quad (o = I) \wedge (j < J^R), \\ \phi^R(\omega)(1+\Delta_{\phi^R,t}) & \text{otherwise}, \end{cases} \tag{A.32}$$

with $\phi^I > 0$ the guaranteed minimum income and $\phi^R(\omega) = \phi^R\omega\frac{\sum_{j\in\boldsymbol{J}} e_j}{J-18}, \phi^R > 0$ the skill-group specific retirement pensions. The instrument $\Delta_{\phi^R,t}$ represents a way for the government to balance its budget constraint by modulating pensions.

A.4.2 Bequests

At the end of period t, a fraction of households dies, including migrants, and their accumulated assets are transferred as bequests to households surviving into next period. The aggregate bequest collected is thus

$$\Xi_t \equiv \int_{\boldsymbol{S}_{t-1}} (1-\psi_{j+1})(1+m_j) \sum_{o\in \boldsymbol{O}} a_t'(s;o) P_t(o \mid s)\lambda_t(ds). \tag{A.33}$$

Let $\mathfrak{B}_t$ denote the aggregate bequests distributed in period t, so that

$$\mathfrak{B}_t = \Xi_{t-1}. \tag{A.34}$$

Alternatively, recall that individual bequests are given by $\xi_t(\omega, j)$, assumed to take the form

$$\xi_t(\omega, j) = \xi_t \xi_\omega \xi_j \tag{A.35}$$

where (ξ_ω, ξ_j) capture the relative bequest redistribution across ability groups and ages, respectively. The term ξ_t ensures the clearing of the bequest account, so that

$$\mathfrak{B}_t = \int_{\boldsymbol{S}_{t-1}} \xi_t(\omega, j)\lambda_t(ds). \tag{A.36}$$

A.5 Government

The government spends public consumption, G_t, makes social transfers to households

$$\text{Tr}_t = \int_{\boldsymbol{S}_{t-1}} \sum_{o\in \boldsymbol{O}} \text{tr}_t(\omega, j, o) P_t(o \mid s)\lambda_t(ds),$$

pays interest on its debt and finances all those expenditures by levying taxes on consumption, labour income, firm's capital stock, and emitting public debt B_t, B_{-1} given, so as to keep the debt:output ratio constant (i.e, $s_b \equiv B_{t-1}/Y_t$). Its budget constraint, therefore, reads as

$$G_t + \text{Tr}_t + (1+r_{b,t})B_{t-1} = \tau_t^C C_t + \mathscr{T}_t^N + \tau_t^K r_{k,t} K_{t-1} + B_t, \tag{A.37}$$

or

$$G_t + \text{Tr}_t + (1+r_{b,t})s_b Y_t = \tau_t^C C_t + \mathscr{T}_t^N + \tau_t^K r_{k,t} K_{t-1} + s_b Y_{t+1}, \tag{A.38}$$

with $C_t \equiv \int_{\boldsymbol{S}_{t-1}} \sum_{o\in \boldsymbol{O}} c_t(s;o) P_t(o \mid s)\lambda_t(ds)$ the aggregate consumption and $\mathscr{T}_t^N$ the labour-income-based fiscal revenues defined by

$$\mathscr{T}_t^N \equiv \int_{\boldsymbol{S}_{t-1}} P_t(o \in \{E\} \mid s) \mathscr{T}_t\left(\bar{h}_j w_t e_j \omega z\right)\lambda_t(ds). \tag{A.39}$$

A.6 Aggregation and Market Clearing Conditions

Total savings offered at the end of period t writes

$$S_t \equiv \int_{\boldsymbol{S}_{t-1}} (1+m_j) \sum_{o \in \boldsymbol{O}} a'_t(s;o) P_t(o \mid s) \lambda_t(ds). \tag{A.40}$$

Hence, financial markets clear so that the demand for assets at the end of period corresponds to the supply of assets to be used in the next period

$$S_t = K_t + B_t.$$

Similarly, labour markets clear so that

$$N_t = \int_{\boldsymbol{S}_{t-1}} P_t(o \in \{E\} \mid s) e_j \omega z \bar{h}_j \lambda_t(ds).$$

Note that if we let A_t denote the total wealth detained by households before receiving bequests at the beginning of period $t+1$, we have

$$A_t \equiv \int_{\boldsymbol{S}_t} a \lambda_{t+1}(ds).$$

Alternatively, at the end of period (i.e., after incoming immigrants and after deaths occur), total weatlh is equivalently given by

$$A_t = \int_{\boldsymbol{S}_{t-1}} \psi_{j+1}(1+m_j) \sum_{o \in \boldsymbol{O}} a'_t(s;o) P_t(o \mid s) \lambda_t(ds). \tag{A.41}$$

Therefore, we have

$$S_t = A_t + \Xi_t. \tag{A.42}$$

A.7 Equilibrium Definition

This Section states the model structure and then offers a definition of the sequential equilibrium.

1. Aggregate production

$$Y_t = Z\big(K_{t-1}\big)^{\theta} \big(\Gamma_t N_t\big)^{1-\theta}. \tag{A.43}$$

2. Demand for capital

$$(1+\tau_t^K) r_{k,t} + \delta = \theta Z \left(\frac{K_{t-1}}{\Gamma_t N_t}\right)^{\theta-1}. \tag{A.44}$$

3. labour demand

$$w_t = (1-\theta) Z \Gamma_t \left(\frac{K_{t-1}}{\Gamma_t N_t}\right)^{\theta}. \tag{A.45}$$

4. Return on public bonds

$$1 + r_{b,t} = (1-\alpha)(1 + r_{k,t}). \tag{A.46}$$

5. Return on assets

$$1 + r_t = (1 + r_{k,t})(1 - \Upsilon_{t-1}), \tag{A.47}$$

with $\Upsilon_t = \alpha B_t / S_t$.

6. Individual problem in state $s = (\omega, j, z, a, o_-)$ for $j < J^R$,

$$V_t(\omega, j, z, a, o_-) = \mathbb{E}_{\chi_t(E),\chi_t(I)}\left[\max_{o\in\{E,I\}}\left\{W_t(\omega, j, z, a; o) + \chi_t(o)\Gamma_t^{1-\sigma}\right\}\right], \tag{A.48}$$

with

$$W_t(\omega, j, z, a; o) = \max_{(c,a')\in\mathcal{H}_t(\omega,j,z,a;o)}\left\{\frac{\left(c + \frac{\Upsilon_t}{1+\tau_t^C}a'\right)^{1-\sigma}}{1-\sigma} - \mathbb{1}[o\in\{E\}]\kappa_{\omega,j}\Gamma_t^{1-\sigma}\right.$$
$$\left. + (1-\psi_{j+1})\nu_a\Gamma_t^{\varphi-\sigma}\frac{\left(\frac{1-\Upsilon_t}{1+\tau_t^C}a'\right)^{1-\varphi}}{1-\varphi} + \beta\psi_{j+1}\sum_{z'\in\boldsymbol{Z}}\Pi^Z(z,z')V_{t+1}(\omega, j+1, z', a', o)\right\}, \tag{A.49}$$

and for $j \geq J^R$

$$V_t(\omega, j, z, a, o_- = E) = \mathbb{E}_{\chi_t(E),\chi_t(I)}\left[\max_{o\in\{E,I\}}\left\{W_t(\omega, j, z, a; o) + \chi_t(o)\Gamma_t^{1-\sigma}\right\}\right], \tag{A.50}$$

with

$$W_t(\omega, j, z, a; o = E) = \max_{(c,a')\in\mathcal{H}_t(\omega,j,z,a;E)}\left\{\frac{\left(c + \frac{\Upsilon_t}{1+\tau_t^C}a'\right)^{1-\sigma}}{1-\sigma} - \kappa_{\omega,j}\Gamma_t^{1-\sigma}\right.$$
$$\left. + (1-\psi_{j+1})\nu_a\Gamma_t^{\varphi-\sigma}\frac{\left(\frac{1-\Upsilon_t}{1+\tau_t^C}a'\right)^{1-\varphi}}{1-\varphi} + \beta\psi_{j+1}\sum_{z'\in\boldsymbol{Z}}\Pi^Z(z,z')V_{t+1}(\omega, j+1, z', a', o = E)\right\}, \tag{A.51}$$

and

$$W_t(\omega, j, z, a; o = I) = W_t^R(\omega, j, a), \text{ for all } z \in \boldsymbol{Z}, \tag{A.52}$$

and

$$V_t(\omega, j, z, a, o_- = I) = W_t^R(\omega, j, a), \ \forall z \in \boldsymbol{Z}, \tag{A.53}$$

with

$$W_t^R(\omega,j,a)=\max_{(c,a')\in\mathcal{H}_t(\omega,j,z,a;I)}\left\{\frac{\left(c+\frac{\Upsilon_t}{1+\tau_t^C}a'\right)^{1-\sigma}}{1-\sigma}\right.$$
$$\left.+(1-\psi_{j+1})\nu_a\Gamma_t^{\varphi-\sigma}\frac{\left(\frac{1-\Upsilon_t}{1+\tau_t^C}a'\right)^{1-\varphi}}{1-\varphi}+\beta\psi_{j+1}W_{t+1}^R(\omega,j+1,a')\right\}. \tag{A.54}$$

and where

$$\mathcal{H}_t(\omega,j,z,a;o)=\Big\{(c,a'):(1+\tau_t^C)c+a'=(1+r_t)\left(a+\xi_t(\omega,j)\right)$$
$$+\operatorname{tr}_t(\omega,j,o)+\mathbb{1}[o\in\{E\}]\left(\bar{h}_je_j\omega z w_t-\mathcal{T}_t(\bar{h}_je_j\omega z w_t)\right);a'\geq 0;c\geq 0\Big\}, \tag{A.55}$$

with the associated choice-specific decision rules $a_t'(\cdot;o)$ and $c_t(\cdot;o)$, and the probability $P_t(o\mid s)$.

7. Markov Transition

$$\begin{aligned}&\forall s=(\omega,j,z,a,o_-)\in\boldsymbol{S}_{t-1}\text{ and }\boldsymbol{S}_{0,t}=\boldsymbol{\Omega}_0\times\boldsymbol{J}_0\times\boldsymbol{Z}_0\times\boldsymbol{A}_{0,t}\times\boldsymbol{O}_0\in\mathcal{B}(\boldsymbol{S}_t),\\&\mathcal{Q}_t(s,\boldsymbol{S}_{0,t})=\mathbb{1}_{\boldsymbol{\Omega}_0}(\omega)\times\psi_{j+1}(1+m_j)\times\mathbb{1}_{\boldsymbol{J}_0}(j+1)\\&\qquad\times\sum_{z'\in\boldsymbol{Z}_0}\Pi^Z(z,z')\times\sum_{o\in\boldsymbol{O}_0}\mathbb{1}_{\boldsymbol{A}_{0,t}}(a_t'(s;o))P_t(o\mid s).\end{aligned} \tag{A.56}$$

8. Law of Motion of Distribution

$$\begin{aligned}&\forall\boldsymbol{S}_{0,t}=\boldsymbol{\Omega}_0\times\boldsymbol{J}_0\times\boldsymbol{Z}_0\times\boldsymbol{A}_{0,t}\times\boldsymbol{O}_0\in\mathcal{B}(\boldsymbol{S}_t)\text{ such that }18\notin\boldsymbol{J}_0,\\&\qquad\lambda_{t+1}(\boldsymbol{S}_{0,t})=\int_{\boldsymbol{S}_{t-1}}\mathcal{Q}_t(s,\boldsymbol{S}_{0,t})\lambda_t(ds),\\&\forall\boldsymbol{S}_{0,t}=\boldsymbol{\Omega}_0\times\{18\}\times\boldsymbol{Z}_0\times\boldsymbol{A}_{0,t}\times\boldsymbol{O}_0\in\mathcal{B}(\boldsymbol{S}_t),\\&\qquad\lambda_{t+1}(\boldsymbol{S}_{0,t})=\sum_{\omega\in\boldsymbol{\Omega}_0}\pi^\Omega(\omega)\times L_{18,t+1}\\&\qquad\qquad\times\sum_{z'\in\boldsymbol{Z}_0}\sum_{z\in\boldsymbol{Z}}\Pi^Z(z,z')\pi^Z(z)\times\sum_{o\in\boldsymbol{O}_0}\sum_{o_-\in\boldsymbol{O}}\mathbb{1}_{\boldsymbol{A}_{0,t}}(0)\pi^O(o_-),\end{aligned} \tag{A.57}$$

where $\pi^O(o)$ is an arbitrary distribution on $\boldsymbol{O}$ since the initial occupational status is irrelevant for newborn agents.

9. Aggregate savings

$$S_t\equiv\int_{\boldsymbol{S}_{t-1}}(1+m_j)\sum_{o\in\boldsymbol{O}}a_t'(s;o)P_t(o\mid s)\lambda_t(ds). \tag{A.58}$$

10. Equilibrium on the capital market

$$K_t + B_t = S_t. \tag{A.59}$$

11. Aggregate labour supply

$$N_t = \int_{\boldsymbol{S}_{t-1}} P_t(o \in \{E\} \mid s) e_j \omega z \bar{h}_j \lambda_t(ds). \tag{A.60}$$

12. Aggregate bequest left

$$\Xi_t \equiv \int_{\boldsymbol{S}_{t-1}} (1 - \psi_{j+1})(1 + m_j) \sum_{o \in \boldsymbol{O}} a_t'(s;o) P_t(o \mid s) \lambda_t(ds). \tag{A.61}$$

13. Aggregate bequest received

$$\mathfrak{B}_t = \int_{\boldsymbol{S}_{t-1}} \xi_t(\omega, j) \lambda_t(ds). \tag{A.62}$$

14. Equilibrium for bequests

$$\Xi_t = \mathfrak{B}_{t+1}. \tag{A.63}$$

15. Aggregate wealth

$$A_t = \int_{\boldsymbol{S}_{t-1}} \psi_{j+1}(1 + m_j) \sum_{o \in \boldsymbol{O}} a_t'(s;o) P_t(o \mid s) \lambda_t(ds). \tag{A.64}$$

16. Equilibrium for wealth

$$S_t = A_t + \Xi_t. \tag{A.65}$$

17. Aggregate transfers

$$\mathrm{Tr}_t = \int_{\boldsymbol{S}_{t-1}} \sum_{o \in \boldsymbol{O}} \left(w_t \phi_t(\omega, j, o) + T_t \right) P_t(o \mid s) \lambda_t(ds). \tag{A.66}$$

18. Aggregate Consumption

$$C_t \equiv \int_{\boldsymbol{S}_{t-1}} \sum_{o \in \boldsymbol{O}} c_t(s;o) P_t(o \mid s) \lambda_t(ds). \tag{A.67}$$

19. labour-Tax Revenues

$$\mathcal{T}_t^N = \int_{\boldsymbol{S}_{t-1}} P_t(o \in \{E\} \mid s) \mathcal{T}_t \left(\bar{h}_j w_t e_j \omega z \right) \lambda_t(ds). \tag{A.68}$$

20. Government Budget Constraint

$$G_t + \mathrm{Tr}_t + (1 + r_{b,t}) s_b Y_t = \tau_t^C C_t + \mathcal{T}_t^N + \tau_t^K r_{k,t} K_{t-1} + s_b Y_{t+1}. \tag{A.69}$$

Armed with this sequential representation, we can now offer the following equilibrium definition.

Definition 1. *Given policy parameters* (s_b, J^R)*, a sequence of government expenditures and lump-sum transfers* $\{G_t, T_t\}_{t=0}^{\infty}$*, an initial distribution of agents* λ_0 *over the state space* $\boldsymbol{S}_{-1}$*, an initial capital stock* K_{-1}*, and an initial productivity level* Γ_0*, a sequential equilibrium is a sequence of prices* $\{r_t, r_{k,t}, r_{b,t}, w_t\}_{t=0}^{\infty}$*, a sequence of received bequests* $\{\xi_t(\cdot)\}_{t=0}^{\infty}$*, a sequence of discount on the return to capital* $\{\Upsilon_t\}_{t=0}^{\infty}$ *a sequence of value functions* $\{V_t(s), W_t(s)\}_{t=0}^{\infty}$ *with an associated sequence of choice-specific decision rules* $\{a'_t(s;o), c_t(s;o)\}_{t=0}^{\infty}$ *and an associated sequence of labour-market participation probabilities* $\{P_t(o \mid s)\}_{t=0}^{\infty}$*, a sequence of fiscal adjustments* $\{\tau_t^C, \tau_t^K, \mathscr{T}_t(\cdot), \phi_t(\cdot)\}_{t=0}^{\infty}$*, a sequence of measures* $\{\lambda_{t+1}\}_{t=0}^{\infty}$ *of agents over the state-spaces* $\{\boldsymbol{S}_t\}_{t=0}^{\infty}$*, and a sequence of aggregate quantities* $\{K_t, N_t, \Xi_t\}_{t=0}^{\infty}$*, such that:*

1. *The value functions solve the individual problems stated in item 6 above, with associated decision rules* $a'_t(s;o), c_t(s;o)$ *and labour-market participation probability* $P_t(o \mid s)$*, implying the Markov transition stated in item 7 above;*

2. *The returns on assets, capital, and public bonds bonds, and the discount on the return on capital obey the relations stated in items 4 and 5;*

3. *Given the price system, firms maximise profits, yielding the first-order conditions stated in items 2 and 3 above;*

4. *The fiscal adjustments ensure the government runs a balanced budget, as in item 20 above;*

5. *Aggregate bequests left* Ξ_t *and aggregate capital* K_t *obey the equilibrium on the capital market conditions stated in items 9, 10, 12, 15, and 16 above;*

6. *Aggregate bequest left obey the equilibrium conditions stated in item 13 and 14 above;*

7. *Aggregate labour obeys the equilibrium labour market condition stated in item 11 above;*

8. *The distribution evolves according to the relations stated in item 8 above.*

A.8 De-trended Model

In the previous formulation of the model, macroeconomic variables exhibit a trend. To stationarise them, each trending variable is divided by its corresponding trend component, and de-trended variables are subsequently marked with "ˆ". In this Section, we focus exclusively on a de-trended version of the above economy where the state space is $\boldsymbol{S}$. We define the stationary variables

$$\hat{Y}_t \equiv \frac{Y_t}{\Gamma_t L_t}, \hat{C}_t \equiv \frac{C_t}{\Gamma_t L_t}, \hat{K}_t \equiv \frac{K_t}{\Gamma_t L_t}, \hat{B}_t \equiv \frac{B_t}{\Gamma_t L_t}, \hat{S}_t \equiv \frac{S_t}{\Gamma_t L_t}, \hat{A}_t \equiv \frac{A_t}{\Gamma_t L_t}, \hat{\Xi}_t \equiv \frac{\Xi_t}{\Gamma_t L_t}, \mathfrak{B}_t \equiv \frac{\mathfrak{B}_t}{\Gamma_t L_t},$$

$$\hat{\mathrm{Tr}}_t \equiv \frac{\mathrm{Tr}_t}{\Gamma_t L_t}, \hat{G}_t \equiv \frac{G_t}{\Gamma_t L_t}, \hat{\mathscr{T}}_t^N \equiv \frac{\mathscr{T}_t^N}{\Gamma_t L_t}, \hat{w}_t \equiv \frac{w_t}{\Gamma_t}, \hat{\mathrm{tr}}_t \equiv \frac{\mathrm{tr}_t}{\Gamma_t}, \hat{T}_t \equiv \frac{T_t}{\Gamma_t}, \hat{\xi}_t \equiv \frac{\xi_t}{\Gamma_t}, \hat{N}_t \equiv \frac{N_t}{L_t}, \hat{\lambda}_t \equiv \frac{\lambda_t}{L_t}.$$

The representation of the model economy in its stationary form becomes

1. Aggregate production

$$\hat{Y}_t = Z\left(\frac{\hat{K}_{t-1}}{(1+g)\hat{N}_t}\right)^{\theta}\hat{N}_t. \tag{A.70}$$

2. Demand for capital

$$(1+\tau_t^K)r_{k,t}+\delta = \theta Z\left(\frac{\hat{K}_{t-1}}{(1+g)\hat{N}_t}\right)^{\theta-1}. \tag{A.71}$$

3. labour demand

$$\hat{w}_t = (1-\theta)Z\left(\frac{\hat{K}_{t-1}}{(1+g)\hat{N}_t}\right)^{\theta} = (1-\theta)Z\left(\frac{(1+\tau^K)r_t^k+\delta}{\theta Z}\right)^{\frac{\theta}{\theta-1}}. \tag{A.72}$$

4. Return on public bonds

$$1+r_{b,t} = (1-\alpha)(1+r_{k,t}). \tag{A.73}$$

5. Return on assets

$$1+r_t = (1+r_{k,t})(1-\Upsilon_{t-1}), \tag{A.74}$$

   with $\Upsilon_t = \alpha\hat{B}_t/\hat{S}_t$.

6. Individual problem in state $s=(\omega,j,z,a,o_-)\in\boldsymbol{S}$ for $j<J^R$,

$$\hat{V}_t(\omega,j,z,a,o_-) = \mathbb{E}_{\chi_t(E),\chi_t(I)}\left[\max_{o\in\{E,I\}}\left\{\hat{W}_t(\omega,j,z,a;o)+\chi_t(o)\right\}\right], \tag{A.75}$$

   with

$$\hat{W}_t(\omega,j,z,a;o) = \max_{(c,a')\in\mathscr{H}_t(\omega,j,z,a;o)}\left\{\frac{\left(c+\frac{\Upsilon_t}{1+\tau_t^C}a'\right)^{1-\sigma}}{1-\sigma} - \mathbb{1}[o\in\{E\}]\kappa_{\omega,j}\right.$$
$$\left. +(1-\psi_{j+1})\nu_a\frac{\left(\frac{1-\Upsilon_t}{1+\tau_t^C}a'\right)^{1-\varphi}}{1-\varphi} + \beta(1+\gamma)^{1-\sigma}\psi_{j+1}\sum_{z'\in\boldsymbol{Z}}\Pi^Z(z,z')\hat{V}_{t+1}(\omega,j+1,z',a',o)\right\}, \tag{A.76}$$

   and for $j\geq J^R$

$$\hat{V}_t(\omega,j,z,a,o_-=E) = \mathbb{E}_{\chi_t(E),\chi_t(I)}\left[\max_{o\in\{E,I\}}\left\{\hat{W}_t(\omega,j,z,a;o)+\chi_t(o)\right\}\right], \tag{A.77}$$

with

$$\hat{W}_t(\omega, j, z, a; o = E) = \max_{(c,a')\in\mathscr{H}_t(\omega,j,z,a;E)} \left\{ \frac{\left(c + \frac{\Upsilon_t}{1+\tau_t^C} a'\right)^{1-\sigma}}{1-\sigma} - \kappa_{\omega,j} \right.$$

$$\left. + (1-\psi_{j+1})\nu_a \frac{\left(\frac{1-\Upsilon_t}{1+\tau_t^C} a'\right)^{1-\varphi}}{1-\varphi} + \beta(1+\gamma)^{1-\sigma}\psi_{j+1} \sum_{z'\in \boldsymbol{Z}} \Pi^Z(z,z')\hat{V}_{t+1}(\omega, j+1, z', a', o = E) \right\}, \quad \text{(A.78)}$$

and

$$\hat{W}_t(\omega, j, z, a; o = I) = \hat{W}_t^R(\omega, j, a), \text{ for all } z \in \boldsymbol{Z}, \quad \text{(A.79)}$$

and

$$\hat{V}_t(\omega, j, z, a, o_- = I) = \hat{W}_t^R(\omega, j, a), \ \forall z \in \boldsymbol{Z}, \quad \text{(A.80)}$$

with

$$\hat{W}_t^R(\omega, j, a) = \max_{(c,a')\in\mathscr{H}_t(\omega,j,z,a;I)} \left\{ \frac{\left(c + \frac{\Upsilon_t}{1+\tau_t^C} a'\right)^{1-\sigma}}{1-\sigma} \right.$$

$$\left. + (1-\psi_{j+1})\nu_a \frac{\left(\frac{1-\Upsilon_t}{1+\tau_t^C} a'\right)^{1-\varphi}}{1-\varphi} + \beta(1+\gamma)^{1-\sigma}\psi_{j+1}\hat{W}_{t+1}^R(\omega, j+1, a') \right\}. \quad \text{(A.81)}$$

and where

$$\mathscr{H}_t(\omega, j, z, a; o) = \Big\{(c, a') : (1+\tau_t^C)c + a' = (1+r_t)\left(a/(1+\gamma) + \hat{\xi}_t \xi_\omega \xi_j\right)$$

$$+ \hat{\text{tr}}_t(\omega, j, o) + \mathbb{1}[o \in \{E\}](1-\tau_t^N)\left(\bar{h}_j \hat{w}_t e_j \omega z\right)^{1-\zeta_t}; a' \geq 0; c \geq 0\Big\}, \quad \text{(A.82)}$$

with the associated choice-specific decision rules $g_{a,t}(\omega, j, z, a; o)$ and $g_{c,t}(\omega, j, z, a; o)$, and the probability $P_t(o \mid s)$.

7. Markov Transition

$\forall s = (\omega, j, z, a, o_-) \in \boldsymbol{S}$ and $\boldsymbol{S}_0 = \boldsymbol{\Omega}_0 \times \boldsymbol{J}_0 \times \boldsymbol{Z}_0 \times \boldsymbol{A}_0 \times \boldsymbol{O}_0 \in \mathscr{B}(\boldsymbol{S})$,

$$\mathscr{Q}_t(s, \boldsymbol{S}_0) = \mathbb{1}_{\boldsymbol{\Omega}_0}(\omega) \times \psi_{j+1}(1+m_j) \times \mathbb{1}_{\boldsymbol{J}_0}(j+1)$$
$$\times \sum_{z'\in \boldsymbol{Z}_0} \Pi^Z(z,z') \times \sum_{o\in \boldsymbol{O}_0} \mathbb{1}_{\boldsymbol{A}_0}(g_{a,t}(s;o))P_t(o \mid s). \quad \text{(A.83)}$$

8. Law of Motion of Distribution

$$\begin{aligned}
&\forall \boldsymbol{S}_0 = \boldsymbol{\Omega}_0 \times \boldsymbol{J}_0 \times \boldsymbol{Z}_0 \times \boldsymbol{A}_0 \times \boldsymbol{O}_0 \in \mathscr{B}(\boldsymbol{S}) \text{ such that } 18 \notin \boldsymbol{J}_0,\\
&\qquad \hat{\lambda}_{t+1}(\boldsymbol{S}_0) = \frac{1}{1+n}\int_{\boldsymbol{S}} \mathscr{Q}_t(s, \boldsymbol{S}_0)\hat{\lambda}_t(ds),\\
&\forall \boldsymbol{S}_0 = \boldsymbol{\Omega}_0 \times \{18\} \times \boldsymbol{Z}_0 \times \boldsymbol{A}_0 \times \boldsymbol{O}_0 \in \mathscr{B}(\boldsymbol{S}),\\
&\qquad \hat{\lambda}_t(\boldsymbol{S}_0) = \mu_{18} \times \sum_{\omega\in\boldsymbol{\Omega}_0} \pi^{\Omega}(\omega) \times \sum_{z\in\boldsymbol{Z}_0} \pi^{Z}(z) \times \mathbb{1}_{\boldsymbol{A}_0}(0) \times \sum_{o\in\boldsymbol{O}_0} \pi^{O}(o),
\end{aligned} \tag{A.84}$$

with $\pi^O(o)$ an arbitrary distribution on $\boldsymbol{O}$.

9. Aggregate savings

$$\hat{S}_t = \int_{\boldsymbol{S}} (1+m_j) \sum_{o\in\boldsymbol{O}} g_{a,t}(s;o) P_t(o \mid s) \hat{\lambda}_t(ds). \tag{A.85}$$

10. Equilibrium on the capital market

$$\hat{K}_t + \hat{B}_t = \hat{S}_t. \tag{A.86}$$

11. Aggregate labour supply

$$\hat{N}_t = \int_{\boldsymbol{S}} P_t(o \in \{E\} \mid s) e_j \omega z \bar{h}_j \hat{\lambda}_t(ds). \tag{A.87}$$

12. Aggregate bequest left

$$\hat{\Xi}_t \equiv \int_{\boldsymbol{S}} (1-\psi_{j+1})(1+m_j) \sum_{o\in\boldsymbol{O}} g_{a,t}(s;o) P_t(o \mid s) \hat{\lambda}_t(ds). \tag{A.88}$$

13. Aggregate bequest received

$$\hat{\mathfrak{B}}_t = \int_{\boldsymbol{S}} \hat{\xi}_t \xi_\omega \xi_j \hat{\lambda}_t(ds). \tag{A.89}$$

14. Equilibrium for bequests

$$\hat{\Xi}_t = \hat{\mathfrak{B}}_{t+1}(1+g). \tag{A.90}$$

15. Aggregate wealth

$$\hat{A}_t = \int_{\boldsymbol{S}} \psi_{j+1}(1+m_j) \sum_{o\in\boldsymbol{O}} g_{a,t}(s;o) P_t(o \mid s) \hat{\lambda}_t(ds). \tag{A.91}$$

16. Equilibrium for wealth

$$\hat{S}_t = \hat{A}_t + \hat{\Xi}_t. \tag{A.92}$$

17. Aggregate transfers

$$\hat{Tr}_t = \int_{\boldsymbol{S}} \sum_{o\in\boldsymbol{O}} \left(\hat{w}_t \phi_t(\omega, j, o) + \hat{T}_t\right) P_t(o \mid s) \hat{\lambda}_t(ds). \tag{A.93}$$

18. Aggregate Consumption

$$\hat{C}_t \equiv \int_{\boldsymbol{S}} \sum_{o\in\boldsymbol{O}} g_{c,t}(s;o) P_t(o \mid s) \hat{\lambda}_t(ds). \tag{A.94}$$

19. Labour-Tax Revenues

$$\hat{\mathcal{T}}_t^N = \int_S P_t(o \in \{E\} \mid s)\mathcal{T}_t\left(\bar{h}_j \hat{w}_t e_j \omega z\right) \hat{\lambda}_t(ds). \tag{A.95}$$

20. Government Budget Constraint

$$\hat{G}_t + \hat{Tr}_t + \frac{1 + r_{b,t}}{1+g}\hat{B}_{t-1} = \tau_t^C \hat{C}_t + \hat{\mathcal{T}}_t^N + \tau_t^K \frac{r_{k,t}}{1+g}\hat{K}_{t-1} + \hat{B}_t. \tag{A.96}$$

B General Solution Method

This section describes the numerical method used to solve the model. The solution proceeds in several steps:

- First, we assign initial values to the minimal set of economic variables required to solve households' optimization problem, namely $r_{k,t}$, $\hat{\mathfrak{B}}_t$, Υ_t, and Φ_t, where Φ_t denotes the (unique) endogenous fiscal instrument. For example, if the government balances its budget through the capital income tax, then $\Phi_t = \tau_t^K$, while all other fiscal instruments (tax rates, social security replacement rates, public consumption, public debt) are held fixed. Given this set of variables, all remaining prices and the cross-sectional distribution of bequests can be immediately inferred.
- Second, the household problem—which features discrete choices—is solved using the endogenous grid method in order to improve computational efficiency.
- Third, conditional on households' decision rules, we compute the distribution over the state space.
- Fourth, households' policy functions and the associated distributions are used to compute aggregate variables, including firms' and government policies. These aggregates must satisfy general equilibrium conditions, such as market-clearing constraints.

Broadly speaking, we iterate on the initial guesses for the endogenous variables until all equilibrium conditions are simultaneously satisfied, either - depending on the nature of the equilibrium we are solving for - in the steady state or along the transition path.

B.1 Computing prices and individual bequests

Given a sequence for $r_{k,t}, \hat{\mathfrak{B}}_t, \Upsilon_t$, together with all fiscal instruments, we use the labour demand condition (A.72) and the asset return equations (A.73) and (A.74) to determine, respectively, the wage rate, w_t, the interest rate on government bonds, $r_{b,t}$, and the return on the market portfolio, r_t.

Since the distribution of bequests received only depends on age and skill group, equation (A.89) can be used to determine the time-varying component of individual bequests,

$$\hat{\xi}_t = \frac{\hat{\mathfrak{B}}_t}{\sum_{j \in \boldsymbol{J}} \mu_j \sum_{\omega \in \boldsymbol{\Omega}} \xi_j \xi_\omega \pi^\Omega(\omega)} \tag{B.1}$$

and therefore the cross-sectional distribution of individual bequests conditional on $\hat{\mathfrak{B}}_t$.

B.2 Computing Decision Rules

In this section, we describe how we solve for individual de-trended decision rules on consumption $g_{c,t}$ and savings $g_{a,t}$, and labour force participation probabilities P_t.

The endogenous variables relevant to the individual decision problem - namely r_t, w_t, $\hat{\xi}_t$, and Υ_t as well as all fiscal instruments - are taken as given.

B.2.1 Statement of the Problem

Recall the individual problem for $j < J^R$ and $s = (\omega, j, z, a, o_-) \in \boldsymbol{S}$:

$$\hat{V}_t(\omega, j, z, a, o_-) = \mathbb{E}_{\chi_t(E), \chi_t(I)} \left[\max_{o \in \{E, I\}} \left\{ \hat{W}_t(\omega, j, z, a; o) + \chi_t(o) \right\} \right],$$

with

$$\hat{W}_t(\omega, j, z, a; o) = \max_{(c,a') \in \mathscr{H}_t(\omega, j, z, a; o)} \left\{ \frac{\left(c + \frac{\Upsilon_t}{1+\tau_t^C} a'\right)^{1-\sigma}}{1-\sigma} - \mathbb{1}[o \in \{E\}] \kappa_{\omega,j} \right.$$
$$\left. + (1-\psi_{j+1}) \nu_a \frac{\left(\frac{1-\Upsilon_t}{1+\tau_t^C} a'\right)^{1-\varphi}}{1-\varphi} + \beta(1+\gamma)^{1-\sigma} \psi_{j+1} \sum_{z' \in \boldsymbol{Z}} \Pi^Z(z, z') \hat{V}_{t+1}(\omega, j+1, z', a', o) \right\},$$

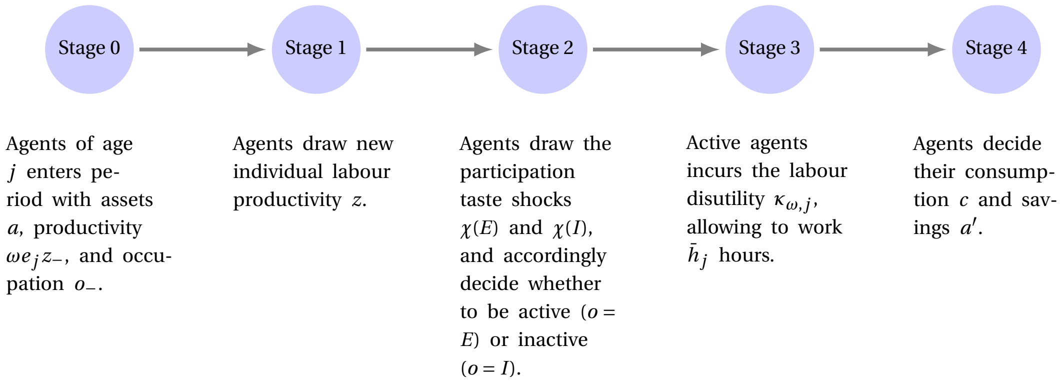
with

$$\mathscr{H}_t(\omega, j, z, a; o) = \Big\{ (c, a') : (1+\tau_t^C) c + a' = (1+r_t)\left(a/(1+\gamma) + \hat{\xi}_t \xi_\omega \xi_j\right)$$
$$+ \hat{\text{tr}}_t(\omega, j, o) + \mathbb{1}[o \in \{E\}] \left(\bar{h}_j \hat{w}_t e_j \omega z - \mathscr{T}_t(\bar{h}_j \hat{w}_t e_j \omega z)\right); a' \geq 0; c \geq 0 \Big\}.$$

Following Bardóczy (2022), it is convenient to divide the period into several stages. Each stage determines a particular household decision conditional on the revelation of the relevant source of uncertainty. The timing of events is summarised in Figure 13 and further described below:

Stage 0. Agent enters the period with individual state vector (ω, j, z_-, a, o_-), i.e., skill group ω, age j, transitory productivity z_-, wealth a and occupation o_-;

Figure 13: Intra-Period Timeline



Stage 1. Transitory productivity is revealed and updated to z. Accordingly, the expected intertemporal utility of a household before transitory productivity is updated, is given by:

$$V_t^0(\omega, j, z_-, a, o_-) = \sum_z \Pi^Z(z_-, z) V_t^1(\omega, j, z, a, o_-). \tag{B.2}$$

Stage 2. In stage 2, the current individual productivity is known. The occupational status of individuals depends on their decision to participate in the labour market. The expected utility is

$$V_t^1(\omega, j, z, a, o_-) = \mathbb{E}_{\chi(E),\chi(I)}\left\{\max\left\{W_t^p(\omega, j, z, a, o_-) + \chi(E), W_t^n(\omega, j, z, a, o_-) + \chi(I)\right\}\right\}. \tag{B.3}$$

Stage 3. Participation/Non-participation: The occupation of an agent who has decided to participate in the labour market is $o = E$. The utility of the participating household is thus given by

$$W_t^p(\omega, j, z, a, o_-) = V_t^3(\omega, j, z, a, E) \tag{B.4}$$

Non-participating agent's value is

$$W_t^n(\omega, j, z, a, o_-) = V_t^3(\omega, j, z, a, I). \tag{B.5}$$

At this stage, it is worth noting that consumption and savings decisions for households that have not yet reached the retirement age do not depend on the occupational status prevailing prior to those decisions, as described by equation (A.76). Consequently, the status o_- is not relevant for the participation decision of this age group, and the dependence on this variable could have been omitted in (B.3).

By contrast, once agents reach the retirement age, inactivity becomes an absorbing state, as characterised by equations (A.77)–(A.81). Participation decisions then concern only agents in state $o_- = E$ and are constrained for those with $o_- = I$. For this reason, and in order to preserve a general formulation, we explicitly retain the dependence on o_- in (B.4) and (B.5), with the understanding that this variable is a state variable with a particular status.

Stage 4. A j-cohort agent who decided to work earns net labour income $\bar{h}_j \hat{w}_t e_j \omega z - \mathscr{T}_t\left(\bar{h}_j \hat{w}_t e_j \omega z\right)$, where $\hat{w}_t$ is the de-trended wage rate, $e_j \omega z$ is individual productivity, and $\mathscr{T}_t$ characterises the labour income tax schedule. Inactive agents receive an age-and-state-specific transfer from the government, $\hat{\mathrm{tr}}_t(\omega, j, o)$ as described in the main text. In addition to labour income and government transfers, agents collect interest r_t on their accumulated capital and bequests received. They simultaneously decide how much to consume, c and how many assets to accumulate, a', subject to a no-borrowing constraint. Therefore, the dynamic problem takes the following form

$$
\begin{aligned}
V_t^3(\omega, j, z, a, o) = \max_{(c,a')} \Bigg\{ & \frac{\left(c + \frac{\Upsilon_t}{1+\tau_t^C} a'\right)^{1-\sigma}}{1-\sigma} - \mathbb{1}[o \in \{E\}]\kappa_{\omega,j} \\
& + (1-\psi_{j+1})\nu_a \frac{\left(\frac{1-\Upsilon_t}{1+\tau_t^C} a'\right)^{1-\varphi}}{1-\varphi} + \Lambda_{j+1} V_{t+1}^0(\omega, j+1, z, a', o) \Bigg\},
\end{aligned}
$$

s.t.

$$
\begin{aligned}
&(1+\tau_t^C)c + a' = (1+r_t)\left(a/(1+\gamma) + \hat{\xi}_t \xi_\omega \xi_j\right) \\
&\qquad + \hat{\mathrm{tr}}_t(\omega, j, o) + \mathbb{1}[o \in \{E\}]\left(\bar{h}_j \hat{w}_t e_j \omega z - \mathscr{T}_t\left(\bar{h}_j \hat{w}_t e_j \omega z\right)\right), \\
&c \geq 0, a' \geq 0,
\end{aligned}
\tag{B.6}
$$

where $\Lambda_{j+1} \equiv \beta(1+\gamma)^{1-\sigma}\psi_{j+1}$.

The associated first-order condition for an interior solution is

$$
EV_{a,t}(\omega, j, z, a', o) = \frac{1-\Upsilon_t}{1+\tau_t^C}\left(\left(c + \frac{\Upsilon_t}{1+\tau_t^C} a'\right)^{-\sigma} - (1-\psi_{j+1})\nu_a \left(\frac{1-\Upsilon_t}{1+\tau_t^C} a'\right)^{-\varphi}\right) \tag{B.7}
$$

where (c, a') are solutions of (B.6) and $EV_{a,t}(\omega, j, z, a', o)$ is the expected marginal value for cohort j at date t:

$$
EV_{a,t}(\omega, j, z, a', o) \equiv \Lambda_{j+1} V_{a,t+1}^0(\omega, j+1, z, a', o), \tag{B.8}
$$

and the envelope condition is

$$
V_{a,t}^3(\omega, j, z, a, o) = \left(c + \frac{\Upsilon_t}{1+\tau_t^C} a'\right)^{-\sigma} \frac{1+r_t}{\left(1+\tau_t^C\right)(1+\gamma)}. \tag{B.9}
$$

B.2.2 Computing the Policy Functions Backward Using DC-EGM

We specify a grid of values for next period's assets $a' \in G_a$, with N_a elements, and we let $G_z = \boldsymbol{Z}$, with N_z elements, which corresponds to a discretization of the temporary idiosyncratic productivity shock. We start by considering agents belonging to the ability group ω and will iterate backward on all the cohorts from $j = J$ to $j = 18$.

Initialization Let consider the cohort j and postulate for the next cohort, namely $j+1$, the values $V^0_{t+1}(\omega, j+1, z, a', o)$ and $V^0_{a,t+1}(\omega, j+1, z, a', o)$ (with $V^0_{t+1}(\omega, J+1, z, a', o) = V^0_{a,t+1}(\omega, J+1, z, a', o) = 0$ for all tuple (t, ω, z, a', o)) from which we compute $EV_t(\omega, j, z, a', o)$ using

$$EV_t(\omega, j, z, a', o) \equiv \Lambda_{j+1} V^0_{t+1}(\omega, j+1, z, a', o). \tag{B.10}$$

and $EV_{a,t}(\omega, j, z, a', o)$ using (B.8). In the remaining part of this Section, we fix ω and j.

Given $V^0_{a,t+1}$ and V^0_{t+1}, we seek $V^0_{a,t}$ and V^0_t to update backward EV_t and $EV_{a,t}$. The procedure described below is used both in steady-state or in a transition context.

For convenience, we let m_t define the cash-on-hand available at the beginning of period t:

$$m_t(\omega, j, z, a, o) \equiv \frac{1+r_t}{1+\gamma} a + (1+r_t)\hat{\xi}_t \xi_\omega \xi_j + \hat{\mathrm{tr}}_t(\omega, j, o) + \mathbb{1}[o \in \{E\}](1-\tau^N_t)\left(\bar{h}_j \hat{w}_t e_j \omega z\right)^{1-\zeta_t}. \tag{B.11}$$

Note that m_t can be computed once and for all on the exogenous grid G_a when r_t, $\hat{w}_t$, ξ_t, and Υ_t as well as all fiscal instruments are known.

Using (B.7), we define c^{egm}_t as the consumption level that a household would have if its individual productivity in period t were z and if it had chosen an asset level for period $t+1$ equal to $a' \in G_a$:

$$c^{egm}_t(\omega, j, z, a', o) = \left(EV_{a,t}(\omega, j, z, a', o) \frac{1+\tau^C_t}{1-\Upsilon_t} + (1-\psi_{j+1})\nu_a \left(\frac{1-\Upsilon_t}{1+\tau^C_t} a' \right)^{-\varphi} \right)^{-\frac{1}{\sigma}} - \frac{\Upsilon_t}{1+\tau^C_t} a' \tag{B.12}$$

Then, given c^{egm}_t, we use the budget constraint to solve for the endogenous cash-on-hand m^{egm}_t:

$$m^{egm}_t(\omega, j, z, a', o) = (1+\tau^C_t) c^{egm}_t(\omega, j, z, a', o) + a'. \tag{B.13}$$

We thus obtain relations $m^{egm}_t(\omega, j, z, a', o) \mapsto c^{egm}_t(\omega, j, z, a', o)$ and $m^{egm}_t(\omega, j, z, a', o) \mapsto a'$. A clarification is warranted at this stage. In cases where the optimality conditions are well-behaved—specifically, when the value function is concave—the mapping from current cash on hand to future assets, $m^{egm}_t \mapsto a'$, is monotonic. In such settings, standard linear interpolation can be used to project the policy function onto the exogenous asset grid. However, when the value function V^0_{t+1} is not concave, the marginal value $V^0_{a,t+1}$ is not well-defined, and the mapping may no longer be a function but rather a correspondence. This situation typically arises when the optimization problem includes discrete choices. In such cases, standard linear interpolation is no longer appropriate, and an upper envelope method must be employed instead.

Monotonic Case. We simply cast the mapping $m_t^{egm}(\omega, j, z, a', o) \mapsto a'$ and the continuation value $m_t^{egm}(\omega, j, z, a', o) \mapsto EV_t(\omega, j, z, a', o)$ on the exogenous cash-on-hand $m_t(\omega, j, z, a, o)$ with $a \in G_a$ using linear interpolation technique. This yields decision rules $\hat{g}_{a,t}(\omega, j, z, a, o)$ and a continuation value $\widehat{EV}_t(\omega, j, z, a, o)$, where the hatted object indicates the fact that we do not take yet the borrowing constraint into account. We thus compute current consumption as

$$\hat{g}_{c,t}(\omega, j, z, a, o) = \frac{1}{1+\tau_t^C}\left(m_t(\omega, j, z, a, o) - \hat{g}_{a,t}(\omega, j, z, a, o)\right) \tag{B.14}$$

and the value function

$$\widehat{V}_t^3(\omega, j, z, a, o) = u_t(\hat{g}_{c,t}(\omega, j, z, a, o), \hat{g}_{a,t}(\omega, j, z, a, o)) + \widehat{EV}_t(\omega, j, z, a, o), \tag{B.15}$$

with

$$u_t(c, a') = \frac{\left(c + \frac{\Upsilon_t}{1+\tau_t^C} a'\right)^{1-\sigma}}{1-\sigma} + (1-\psi_{j+1})\nu_a \frac{\left(\frac{1-\Upsilon_t}{1+\tau_t^C} a'\right)^{1-\varphi}}{1-\varphi}.$$

Upper-Envelope Case. When the mapping $m_t^{egm}(\omega, j, z, a', o) \mapsto a'$ no longer defines a function but rather a correspondence—typically when the value function is non-differentiable due to the presence of discrete choices —we rely on the upper envelope method as described by Iskhakov et al. (2017). In broad terms, the method involves selecting, from the set of interpolated candidates, the one that delivers the highest utility value.

Given (ω, j), for each $(z, a, o) \in G_z \times G_a \times \{E, I\}$, we start by initializing $\widehat{V}_t^3(\omega, j, z, a, o) = -\infty$ and defining $\mathfrak{m}_k = m_t^{egm}(\omega, j, z, a_k, o)$, for $a_k \in G_a$. The point $\mathfrak{m}_k$ refers to the k^{th} element of the endogenous grid, which corresponds to an optimal savings decision equals to $a_k \in G_a$. We then consider each interval defined by two consecutive elements of the endogenous grid $\mathfrak{I}_k = [\mathfrak{m}_k, \mathfrak{m}_{k+1}]$. Note that since the mapping can be a correspondence, we do not necessarily have $\mathfrak{m}_k < \mathfrak{m}_{k+1}$. We hence disregard all cases for which $\mathfrak{m}_k$ is greater than $\mathfrak{m}_{k+1}$. (These cases signal a discontinuity in the mapping.)

Then, for each interval $\mathfrak{I}_k$, we screen all element $m_i = m_t(\omega, j, z, a_i, o)$ with $a_i \in G_a$. If the grid point m_i lies inside the interval, $m_i \in \mathfrak{I}_k$, then we can linearly interpolate the mapping and compute $a'_{new}(i)$. For the last segment $\mathfrak{I}_{N_a-1}$, if the grid point is outside the interval (hence above the highest endogenous grid point), we perform a linear extrapolation.

For each interpolated value $a'_{new}(i)$ on the grid point a_i, we compute (also by interpolation) the continuation value $EV_{new}(i)$ and consumption

$$c_{new}(i) = \frac{1}{1+\tau_t^C}\left(m_i - a'_{new}(i)\right), \tag{B.16}$$

and the value function

$$V_{new}(i) = u_t\left(c_{new}(i), a'_{new}(i)\right) + EV_{new}(i).$$

The resulting value is then compared with those obtained on earlier segments, and the new values are kept whenever they improve upon the existing value function (i.e., $V_{new}(i) \geq \widehat{V_t}^3(\omega, j, z, a_i, o)$) :

$$\hat{g}_{c,t}(\omega, j, z, a_i, o) \leftarrow c_{new}(i), \quad \hat{g}_{a,t}(\omega, j, z, a_i, o) \leftarrow a'_{new}(i),$$

and $\widehat{V_t}^3(\omega, j, z, a, o) \leftarrow V_{new}(i)$. The procedure continues for all subsequent points on the exogenous grid, and is then repeated over the remaining intervals $\mathfrak{I}_k$.

Enforcing the Borrowing Constraint. Note it is possible that for some $(\omega, j, z, a, o) \in \boldsymbol{S}$, $\hat{g}_{a,t}(\omega, j, z, a, o) < 0$. Formally, let us define

$$\mathscr{C}(\omega, j, o) = \{(a, z) \in G_a \times G_z : \hat{g}_{a,t}(\omega, j, z, a, o) < 0\}.$$

For (a, z) in the complement of $\mathscr{C}(\omega, j, o)$, we have

$$g_{c,t}(\omega, j, z, a, o) = \hat{g}_{c,t}(\omega, j, z, a, o), \quad g_{a,t}(\omega, j, z, a, o) = \hat{g}_{a,t}(\omega, j, z, a, o), \quad \widehat{V_t}^3(\omega, j, z, a, o) = \widehat{V_t}^3(\omega, j, z, a, o).$$

In contrast, for $(a, z) \in \mathscr{C}(\omega, j, o)$, we set $g_{a,t}(\omega, j, z, a, o) = 0$. We then have directly

$$g_{c,t}(\omega, j, z, a, o) = \frac{m_t(\omega, j, z, a, o)}{1 + \tau_t^C}. \tag{B.17}$$

We repeat this procedure for each $(a, z) \in \mathscr{C}$. We can then define for these constrained agents the consumption $c_t^*(\omega, j, z, a, o)$ and the value function

$$\left(V_t^3\right)^*(\omega, j, z, a, o) = u_t\left(c_t^*(\omega, j, z, a, o), 0\right) + EV_t(\omega, j, z, 0, o).$$

Therefore, for all $(a, z) \in \mathscr{C}$, we set

$$g_{c,t}(\omega, j, z, a, o) = c_t^*(\omega, j, z, a, o), \quad g_{a,t}(\omega, j, z, a, o) = 0, \quad V_t^3(\omega, j, z, a, o) = \left(V_t^3\right)^*(\omega, j, z, a, o).$$

Updating the Marginal Utility of Wealth. The above procedure allowed us to determine the decision rules $g_{c,t}$ and $g_{a,t}$. We can then use the envelope condition (B.9) to update the derivative of the value function

$$V_{a,t}^3(\omega, j, z, a, o) = \left(g_{c,t}(\omega, j, z, a, o) + \frac{\Upsilon_t}{1 + \tau_t^C} g_{a,t}(\omega, j, z, a, o)\right)^{-\sigma} \frac{1 + r_t}{\left(1 + \tau_t^C\right)(1 + \gamma)}.$$

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The procedure described in Section B.2.2 is then repeated for all admissible values of $(z, o) \in \boldsymbol{Z} \times \boldsymbol{O}$.

B.2.3 Sequential Resolution of Occupational Status and Productivity State

Occupational Status The computation of V_t^3 (i.e., the value function at stage 3 of the general optimization problem) allows calculating and comparing the values of participating or not in the labour market, using (B.4) and (B.5). Regardless of the individual's initial occupational status o_-, agents below the retirement age J^R who choose to participate in the labour market are in state $o \in \{E\}$. However, for those above this age, only individuals who were participating in the previous period (i.e., $o_- = E$) are given the possibility to participate.

This implicitly defines a transition matrix Π_j^p between the initial state o_- and the resulting state o for a participating agent:

$$\Pi_j^p = \begin{cases} \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} & \text{for } j < J^R, \\ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & \text{otherwise,} \end{cases} \tag{B.18}$$

where the rows correspond to the intermediate status o_-, ordered as E and I, and the columns to the resulting status o, in the same order.

Similarly, regardless of the intermediate occupational status, a non-participating agent transitions directly to inactivity. To preserve a consistent structure and facilitate computational implementation, we define the corresponding transition matrix for non-participation as $\Pi_j^n = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$ for all j.

These definitions facilitate the computation of W_t^p and W_t^n at stage 3 allowing a compact matrix representation. Specifically, let V_j denote the $2 \times N_a$ matrix whose entries are given by $\mathsf{V}_j(a,o) = V_t^3(\omega, j, z, a, o)$ for a fixed tuple (ω, j, z). Similarly, define the matrices $\mathsf{W}j^p$ and $\mathsf{W}j^n$, both of dimension $2 \times N_a$, with entries given by $\mathsf{W}^p(a, o_-) = W_t^p(\omega, j, z, a, o_-)$ and $\mathsf{W}^n(a, o_-) = W_t^n(\omega, j, z, a, o_-)$, respectively. We then directly obtain the following expressions:

$$\mathsf{W}_j^p = \mathsf{V}_j \left(\Pi_j^p\right)' - \kappa_{\omega,j}, \tag{B.19}$$

$$\mathsf{W}_j^n = \mathsf{V}_j \left(\Pi_j^n\right)', \tag{B.20}$$

where the scalar $\kappa_{\omega,j}$ is subtracted element-wise.

Proceeding in the same way for all values of z allows us to compute the value functions $W_t^p(\omega, j, z, a, o_-)$ and $W_t^n(\omega, j, z, a, o_-)$ entirely.

Participation Choice. The structure of the problem described in (B.3) allows for an analytical expression of both $p_t(o \mid \omega, j, z, a, o_-)$ defined in (A.27) and V_t^1. In particular, when $\chi(E)$ and $\chi(I)$ are two random variables following a Gumbel distribution $G(-\boldsymbol{\Gamma}\sigma_\chi, \sigma_\chi)$ with $\sigma_\chi > 0$ a scale parameter and $\boldsymbol{\Gamma}$ the Euler–Mascheroni constant, the participation probability and the expectation $\mathbb{E}_{\chi(E),\chi(I)}$ admit the following closed-form expressions. We then obtain

$$p_t(o = E \mid \omega, j, z, a, o_-) = \frac{\exp\left(\dfrac{W_t^p(\omega, j, z, a, o_-)}{\sigma_\chi}\right)}{\exp\left(\dfrac{W_t^p(\omega, j, z, a, o_-)}{\sigma_\chi}\right) + \exp\left(\dfrac{W_t^n(\omega, j, z, a, o_-)}{\sigma_\chi}\right)} \tag{B.21}$$

and

$$V_t^1(\omega, j, z, a, o_-) = \sigma_\chi \log\left(\exp\left(\frac{W_t^p(\omega, j, z, a, o_-)}{\sigma_\chi}\right) + \exp\left(\frac{W_t^n(\omega, j, z, a, o_-)}{\sigma_\chi}\right)\right). \tag{B.22}$$

In practice, we define

$$\Delta V \equiv \frac{W_t^n(\omega, j, z, a, o_-) - W_t^p(\omega, j, z, a, o_-)}{\sigma_\chi} \tag{B.23}$$

and the previous equations are rewritten as follows:

$$p_t(o = E \mid \omega, j, z, a, o_-) = \frac{1}{1 + \exp(\Delta V)} \tag{B.24}$$

and

$$V_t^1(\omega, j, z, a, o_-) = W_t^p(\omega, j, z, a, o_-) - \sigma_\chi \log(p_t(o = E \mid \omega, j, z, a, o_-)). \tag{B.25}$$

Note that from a computational perspective, it is possible for p_t to be smaller than machine precision, which would result in an undefined value for V_t^1 due to the log. Under such circumstances, we use the transformation

$$V_t^1(\omega, j, z, a, o_-) = W_t^n(\omega, j, z, a, o_-) - \sigma_\chi \log\left(1 - p_t(o = E \mid \omega, j, z, a, o_-)\right), \tag{B.26}$$

as long as $\log(p_t) \to -\infty$.

Updating Marginal Values. The previous operations allow us to update the marginal values. More specifically, let $\mathsf{V}_{a,j}$ denote the $N_a \times 2$ matrix whose entries are given by $\mathsf{V}_{a,j}(a,o) = V_{a,t}^3(\omega, j, z, o, a)$ for a fixed tuple (ω, j, z). Similarly, define the matrices $\mathsf{W}_{a,j}^p$ and $\mathsf{W}_{a,j}^n$, both of dimension $N_a \times 2$, with entries given by $\mathsf{W}_{a,j}^p(a, o_-) = W_{a,t}^p(\omega, j, z, a, o_-)$ and $\mathsf{W}_{a,j}^n(a, o_-) = W_{a,t}^n(\omega, j, z, a, o_-)$, respectively, where $W_{a,t}^p$ and $W_{a,t}^n$ are the corresponding marginal values of W_t^p and W_t^n with respect to assets a. Therefore, we obtain

$$\mathsf{W}_{a,j}^p = \mathsf{V}_{a,j}\left(\Pi_j^p\right)', \quad \mathsf{W}_{a,j}^n = \mathsf{V}_{a,j}\left(\Pi_j^n\right)', \tag{B.27}$$

and we finally get

$$\begin{aligned} V_{a,t}^1(\omega, j, z, a, o_-) = {} & P_t(o \in \{E\} \mid \omega, j, z, a, o_-) W_{a,t}^p(\omega, j, z, a, o_-) \\ & + \left(1 - P_t(o \in \{E\} \mid \omega, j, z, a, o_-)\right) W_{a,t}^n(\omega, j, z, a, o_-). \end{aligned} \tag{B.28}$$

Computation of Expectations and Updating. The last step consists in computing the expectations of stage 1 which are directly given by (B.2):

$$V_t^0(\omega, j, z_-, a, o_-) = \sum_z \Pi^Z(z_-, z) V_t^1(\omega, j, z, a, o_-),$$

and for the marginal value

$$V_{a,t}^0(\omega, j, z_-, a, o_-) = \sum_z \Pi^Z(z_-, z) V_{a,t}^1(\omega, j, z, a, o_-). \tag{B.29}$$

From these, we can update $EV_{t-1}(\omega, j, z_-, a, o_-)$ and $EV_{a,t-1}(\omega, j, z_-, a, o_-)$ backward using (B.10) and (B.8).

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The overall procedure is then repeated backward for all $j \in \boldsymbol{J}$ and $\omega \in \boldsymbol{\Omega}$.

B.3 Computing Distributions

In this Section, we detail how we solve for the distributions $\hat{\lambda}_t$.

B.3.1 Distribution by Cohort and Chosen Occupation

To begin with, it may be convenient to make cohort sizes μ_j explicit in the computation of distributions, insofar as these sizes are considered exogenous. To do so, we define the marginal distribution of $\hat{\lambda}_t$ on $\boldsymbol{J}$ by

$$\mu_j(\{j\}) \equiv \hat{\lambda}_t(\boldsymbol{\Omega} \times \{j\} \times \boldsymbol{Z} \times \boldsymbol{A} \times \boldsymbol{O}) = \mu_j,$$

which gives a discrete probability measure over the age set. Then, for each j, we define a conditional measure $\varrho_t(d\omega, dz, da, do_-; j)$ on $\boldsymbol{\Omega} \times \boldsymbol{Z} \times \boldsymbol{A} \times \boldsymbol{O}$ such that

$$\hat{\lambda}_t(d\omega, dj, dz, da, do_-) = \varrho_t(d\omega, dz, da, do_-; j)\mu_j(dj).$$

Now, for each age j, we define a new measure $f_{j,t}$ on $\boldsymbol{S/J} \equiv \boldsymbol{\Omega} \times \boldsymbol{Z} \times \boldsymbol{A} \times \boldsymbol{O}$:

$$f_{j,t}(d\omega, dz, da, do) \equiv \int_{o_- \in \boldsymbol{O}} P_t(do \mid \omega, j, z, a, o_-)\varrho_t(d\omega, dz, da, do_-; j).$$

Thus, the law of motion for the $f_{j,t}$'s replaces that of $\hat{\lambda}_t$, and making use of (A.2), we have for all $j \in \boldsymbol{J}/\{18\}$ and $(\omega, z, a, o) \in \boldsymbol{S/J}$ and $\boldsymbol{S/J}_0 = \boldsymbol{\Omega}_0 \times \boldsymbol{Z}_0 \times \boldsymbol{A}_0 \times \boldsymbol{O}_0 \in \mathscr{B}(\boldsymbol{S/J})$

$$f_{j+1,t+1}(\boldsymbol{S/J}_0) = \int_{\boldsymbol{S/J}} \mathscr{Q}_{j,t}(s, \boldsymbol{S/J}_0) f_{j,t}(ds), \tag{B.30}$$

with

$$\mathscr{Q}_{j,t}((\omega,z,a,o),\boldsymbol{S}/\boldsymbol{J}_0) = \mathbb{1}_{\boldsymbol{\Omega}_0}(\omega)\times\mathbb{1}_{\boldsymbol{A}_0}(g_{a,t}(\omega,j,z,a;o)) \\ \times\sum_{z'\in\boldsymbol{Z}_0}\Pi^Z(z,z')\times\sum_{o'\in\boldsymbol{O}_0}P_{t+1}(o'\mid\omega,j+1,z',g_{a,t}(\omega,j,z,a;o),o), \tag{B.31}$$

and

$$f_{18,t+1}(\boldsymbol{S}/\boldsymbol{J}_0) = \mathbb{1}_{\boldsymbol{\Omega}_0}(\omega)\times\mathbb{1}_{\boldsymbol{A}_0}(0)\times\sum_{o\in\boldsymbol{O}}\sum_{z\in\boldsymbol{Z}}\left(\sum_{z'\in\boldsymbol{Z}_0}\Pi^Z(z,z')\times\sum_{o'\in\boldsymbol{O}_0}P_{t+1}(o'\mid\omega,18,z',0,o)\pi^Z(z)\pi^\Omega(\omega)\pi^O(o)\right), \tag{B.32}$$

with $\pi^O(o)$ an arbitrary distribution on $\boldsymbol{O}$.

B.3.2 Method Approximation and Initialization

Next, we want to compute an approximation of the distribution $f_{j+1,t+1}$ starting from a given distribution $f_{j,t}$. In what follows, the ability group ω will be treated as fixed, and all references to this variable will be omitted whenever this does not lead to confusion. Because, we are focusing on a discrete state space $G_z\times G_a\times\{E,I\}$, our approximation will be interpreted as an array $\mathsf{f}_{j+1,t+1}$ with element $\mathsf{f}_{j+1,t+1}(z,a,o)$ corresponding to the mass of agents at the time of asset and consumption decisions in period $t+1$ with individual productivity $z\in G_z$, assets $a\in G_a$, and occupational status $o\in\{E,I\}$. To compute $\mathsf{f}_{j+1,t+1}$, we use the method proposed by Young (2010).

Specifically, given $(z,a,o)\in G_z\times G_a\times\{E,I\}$, we find the two consecutive grid points $a_k,a_{k+1}\in G_a$, or equivalently the integer $k_t(z,a,o)$ such that

$$a_{k_t(z,a,o)}\le g_{a,t}(\omega,j,z,a;o)\le a_{k_t(z,a,o)+1}, \tag{B.33}$$

letting $k_t(z,a,o)=N_a$ if $g_{a,t}(\omega,j,z,a;o)>\bar{a}$.[19] In this boundary case we extrapolate using the last segment.

Then, given $(z,a,o)\in G_z\times G_a\times\{E,I\}$, we assign a fraction of $g_{a,t}(\omega,j,z,a;o)$ to $a_{k_t(z,a,o)}$ and the remaining fraction to $a_{k_t(z,a,o)+1}$. To this end, we define

$$\varpi_t(z,a,o)=\frac{a_{k_t(z,a,o)+1}-g_{a,t}(\omega,j,z,a;o)}{a_{k_t(z,a,o)+1}-a_{k_t(z,a,o)}}. \tag{B.34}$$

Initialization with the First Generation. We assume prior to birth, the first generation will optimally choose zero asset, what ever the individual state considered (z_-,a,o_-). This allows us to identify $k_t(z_-,a,o_-)$ and $\varpi_t(z_-,a,o_-)$ from (B.33) and (B.34) setting $g_{a,t}=0$.

[19] Notice that different (z_-,a,o_-) can yield the same $k_t(z_-,a,o_-)$.

We initialise our algorithm with an initial distribution $\mathsf{f}_{18,t}$ such that

$$\mathsf{f}_{18,t}(z_-, a, o_-) = 0.$$

We then proceed to iterate over the entire state space $G_z \times G_a \times \{E, I\}$ for the first generation, and carry out the following assignments:

$$\mathsf{f}_{18,t}(z_-, k_t(z_-, a, o_-), o_-) \leftarrow \mathsf{f}_{18,t}(z_-, k_t(z_-, a, o_-), o_-) + \varpi_t(z_-, a, o_-)\pi^{\text{new}}(\omega, z_-, o_-),$$

$$\mathsf{f}_{18,t}(z_-, k_t(z_-, a, o_-) + 1, o_-) \leftarrow \mathsf{f}_{18,t}(z_-, k_t(z_-, a, o_-) + 1, o_-) + (1 - \varpi_t(z_-, a, o_-))\pi^{\text{new}}(\omega, z_-, o_-),$$

with

$$\pi^{\text{new}}(\omega, z_-, o_-) = \pi^{\Omega}(\omega)\pi^{Z}(z_-)\pi^{O}(o_-).$$

The probability $\pi^{\text{new}}(\omega, z_-, o_-)$ represents the unconditional probability for the first generation to start the period t in state (ω, z_-, o_-).

Updating Productivity and Occupational Status for the First Generation. Updating productivity shock from z_- to z is given by

$$\mathsf{f}_{18,t}(z, a, o_-) = \sum_{z_- \in \boldsymbol{Z}} \Pi^{Z}(z_-, z)\mathsf{f}_{18,t}(z_-, a, o_-). \tag{B.35}$$

Similarly, updating the occupational status from o_- to o leads to

$$\mathsf{f}_{18,t}(z, a, o) = \sum_{o_- \in \{E,I\}} P_t(o \mid \omega, 18, z, a, o_-)\mathsf{f}_{18,t}(z, a, o_-), \tag{B.36}$$

with P_t given in (A.28).

B.3.3 Computing Forward the Distribution

The previous Section explains how to compute the distribution for the first cohort at date t. The computation for the following cohorts proceeds the same way. Specifically, we assume that we know $f_{j-1,t-1}$ and we already computed the decision rule $g_{a,t-1}(\omega, j, z, a; o)$. Then, we can compute the corresponding $k_{t-1}(z_-, a, o_-)$ and the $\varpi_{t-1}(z_-, a, o_-)$ from (B.33) and (B.34).

We then iterate over the entire state space $G_z \times G_a \times \{E, I\}$ and carry out the following assignments:

$$\begin{aligned}&\mathsf{f}_{j,t}(z_-, k_{t-1}(z_-, a, o_-), o_-)\\ &\qquad \leftarrow \mathsf{f}_{j,t}(z_-, k_{t-1}(z_-, a, o_-), o_-) + \varpi_{t-1}(z_-, a, o_-)\mathsf{f}_{j-1,t-1}(z_-, a, o_-),\end{aligned}$$

$$\begin{aligned}&\mathsf{f}_{j,t}(z_-, k_{t-1}(z_-, a, o_-) + 1, o_-)\\ &\qquad \leftarrow \mathsf{f}_{j,t}(z_-, k_{t-1}(z_-, a, o_-) + 1, o_-) + [1 - \varpi_{t-1}(z_-, a, o_-)]\mathsf{f}_{j-1,t-1}(z_-, a, o_-).\end{aligned}$$

Finally, updating the productivity shock from z_- to z and updating the occupational status from o_- to o leads to

$$\mathsf{f}_{j,t}(z,a,o_-) = \sum_{z_-\in \boldsymbol{Z}} \Pi^Z(z_-,z)\mathsf{f}_{j,t}(z_-,a,o_-), \tag{B.37}$$

and hence,

$$\mathsf{f}_{j,t}(z,a,o) = \sum_{o_-\in\{E,I\}} P_t(o;\omega,j,z,a,o_-)\mathsf{f}_{j,t}(z,a,o_-). \tag{B.38}$$

By iterating this procedure from $j = 19$ to $j = J$, we obtain the full distribution $\{f_{j,t}\}_{j\in\boldsymbol{J}}$, thereby completing the knowledge about $\hat{\lambda}_t$.

B.4 Computing Aggregates and Residuals

B.4.1 Aggregates

Having computed decision rules and distributions, all is set to compute aggregates that lead to equilibrium conditions. Let μ_j^{mig} define the share of migrants in the j-cohort's population in proportion to the overall population, and μ_j^{tot} the total size of this cohort (i.e., including migration), so that

$$\mu_j^{\text{mig}} \equiv \frac{M_{j,t}}{L_t} = m_j\mu_j \tag{B.39}$$

and

$$\mu_j^{\text{tot}} \equiv \mu_j + \mu_j^{\text{mig}} = \mu_j(1+m_j). \tag{B.40}$$

Let us define $g_{x,t}^j(\omega,z,a;o) \equiv g_{x,t}(\omega,j,z,a;o)$ the j-specific decision rule for $x\in\{c,a,h\}$, and where we note $g_{h,t}^j(\omega,z,a;o) = \mathbb{1}[o\in\{E\}]\bar{h}_j$. Note also that using Fubini's Theorem, for any integrable function $g(s;o)$, we have

$$\int_{\boldsymbol{S}} \sum_{o\in\boldsymbol{O}} g(\omega,j,z,a;o)P_t(o\mid s)\hat{\lambda}_t(ds) = \sum_{j\in\boldsymbol{J}}\mu_j\int_{\boldsymbol{S}/\boldsymbol{J}} g(\omega,j,z,a;o)\, f_{j,t}(d\omega,dz,da,do).$$

Therefore, we have the following aggregates

- Aggregate consumption of period t:

$$\hat{C}_t \equiv \sum_{j\in\boldsymbol{J}}\mu_j\int_{\boldsymbol{S}/\boldsymbol{J}} g_{c,t}^j(s)f_{j,t}(ds).$$

- Aggregate savings supplied:

$$\hat{S}_t = \sum_{j\in\boldsymbol{J}}\mu_j^{\text{tot}}\int_{\boldsymbol{S}/\boldsymbol{J}} g_{a,t}^j(s)f_{j,t}(ds).$$

- Total hours worked:

$$H_t \equiv \sum_{j\in\boldsymbol{J}}\mu_j\int_{\boldsymbol{S}/\boldsymbol{J}} g_{h,t}^j(s)f_{j,t}(ds).$$

- Total transfers:
$$\overline{Tr}_t = \hat{w}_t \sum_{j\in \boldsymbol{J}} \mu_j \int_{\boldsymbol{S/J}} \phi(\omega, j, o) f_{j,t}(ds) + \hat{T}_t.$$
- Aggregate labour supply:
$$\hat{N}_t = \sum_{j\in \boldsymbol{J}} \mu_j \int_{\boldsymbol{S/J}} e_j \omega z g^j_{h,t}(s) f_{j,t}(ds).$$
- Aggregate bequest left:
$$\hat{\Xi}_t = \sum_{j\in \boldsymbol{J}} (1-\psi_{j+1,t}) S_{j,t},$$
with
$$S_{j,t} \equiv \mu^{\text{tot}}_j \int_{\boldsymbol{S/J}} g^j_{a,t}(s) f_{j,t}(ds).$$
- Aggregate wealth
$$\hat{A}_t = \sum_{j\in \boldsymbol{J}} \psi_{j+1,t} \mu^{\text{tot}}_j \int_{\boldsymbol{S}} g^j_{a,t}(s) f_{j,t}(ds).$$

Finally, given the rental rate of capital $r_{k,t}$ and the supply of effective labour, $\hat{N}_t$, we use the capital demand condition (A.71) to determine the aggregate capital stock, $\hat{K}_t$, and the production function (A.29) to determine the level of output, $\hat{Y}_t$.

B.4.2 Equilibrium Conditions

Recall that decision rules, distributions, and aggregates were computed conditional on given values of $r_{k,t}, \hat{\mathfrak{B}}_t, \Upsilon_t, \Phi_t$. We now derive the equilibrium market-clearing conditions that jointly determine these variables in equilibrium.

First, the financial markets must clear in every period, which requires

$$\hat{K}_{t-1} + \hat{B}_{t-1} = \hat{S}_{t-1}.$$

We therefore define the excess demand function for assets as

$$E_{asset}(\{r_{k,t}, \hat{\mathfrak{B}}_t, \Upsilon_t, \Phi_t\}) \equiv \left(\hat{S}_{t-1} - \hat{K}_{t-1} - \hat{B}_{t-1}\right)/\overline{Y}, \tag{B.41}$$

where $\overline{Y}$ is a reference value for production (e.g., the steady state value of $\hat{Y}_t$) as a normalization of the residual.

Second, bequests received at the beginning of period t must equal the bequests left by households in period $t-1$, implying
$$\Xi_{t-1} = \mathfrak{B}_t(1+g).$$

Using (A.92), this condition can be rewritten as the following excess bequest function

$$E_{bequest}(\{r_{k,t},\hat{\mathfrak{B}}_t,\Upsilon_t,\Phi_t\}) \equiv \left(\hat{\mathfrak{B}}_t + \frac{\hat{A}_{t-1}}{1+g} - \frac{\hat{S}_{t-1}}{1+g}\right)/\overline{Y}. \tag{B.42}$$

Intuitively, total assets accumulated during period $t-1$, $\hat{S}_{t-1}$, must be fully transmitted to households alive in period t, either because they survive and retain ownership of their assets, $\hat{A}_{t-1}$, or through bequests, $\hat{\mathfrak{B}}_t$.

Third, we verify that the assumed household portfolio composition is consistent with financial markets equilibrium:

$$E_{portfolio}(\{r_{k,t},\hat{\mathfrak{B}}_t,\Upsilon_t,\Phi_t\}) \equiv (\alpha B_t - \Upsilon_t S_t)/\overline{Y}. \tag{B.43}$$

And fourth, the government budget constraint must hold:

$$E_{govbc}(\{r_{k,t},\hat{B}_t,\Upsilon_t,\Phi_t\}) \equiv \left(\tau_t^C\hat{C}_t + \hat{\mathscr{T}}_t^N + \tau_t^K\frac{r_{k,t}}{1+g}\hat{K}_{t-1} + \hat{B}_t - \hat{G}_t - \hat{Tr}_t - \frac{1+r_{b,t}}{1+g}\hat{B}_{t-1}\right)/\overline{Y}. \tag{B.44}$$

A sequence $\{r_{k,t},\hat{\mathfrak{B}}_t,\Upsilon_t,\Phi_t\}$ is an equilibrium of the model if and only if it simultaneously sets the four residual functions derived above to zero in every period.

Note that, conditional on all other fiscal instruments (taxes, transfers, public debt), public consumption (G_t) does not affect households nor firms behaviors as it does not enter either preferences or the production function. Thus, if the government uses public consumption to balance its budget, $\Phi_t = G_t$, (B.44) can be used to compute $\{G_t\}$ as a residual after having solved the three other residual functions for $\{r_{k,t},\hat{\mathfrak{B}},\Upsilon_t\}$: the sequence of public consumption does not affect any other equilibrium condition.

C Steady State

C.1 Computation

The computation of the steady state relies on a time-invariant version of the model, obtained by removing the time index from all equations and procedures described in Section B.

There is no substantive difference in the procedure depending on whether public spending and public debt are expressed in levels or as ratios to GDP. The steady state can therefore be computed either by fixing the ratios s_B and $s_G \equiv G/Y$ as parameters, or by specifying directly the levels G^* and B^*.

$$\hat{G} = G^* + s_G\hat{Y}, \tag{C.1}$$

and

$$\hat{B} = B^* + s_B(1+g)\hat{Y} \tag{C.2}$$

Of course, if G is the instrument chosen by the government to balance its budget, we simply drop the first equation as the level of G is defined by (B.44)

We begin by postulating values for r_k, B, Υ, and the relevant fiscal instrument Φ. Given these values, we compute the stationary household decision rules using the procedure described in Section B.2, starting from terminal value functions equal to zero ($V^0(\omega, J+1, z, q', o, a') = V_a^0(\omega, J+1, z, q', o, a') = 0$), reflecting the fact that households face a finite maximum lifespan of J periods. We then compute the corresponding stationary distributions as described in Section B.3, form aggregates and finally deduce the vector of residuals $\{E_{asset}(r_k, \hat{\mathfrak{B}}, \Upsilon, \Phi), E_{bequest}(r_k, \hat{\mathfrak{B}}, \Upsilon, \Phi), E_{portfolio}(r_k, \hat{\mathfrak{B}}, \Upsilon, \Phi), E_{gbc}(r_k, \hat{\mathfrak{B}}, \Upsilon, \Phi)\}$ as described in Section B.4.

A numerical solver is then used to drive these residuals sufficiently close to zero, thereby determining values of r_k, $\hat{\mathfrak{B}}$, Υ, and Φ — and hence decision rules, distributions, and aggregates — that satisfy all steady-state equilibrium conditions listed in Section A.8.

C.2 Calibration on Targeted Moments

This Section describes the calibration strategy for the internally calibrated parameters. Conditional on the externally calibrated parameters (see main text), we choose the remaining parameters so that the model's initial steady state matches a set of targeted empirical moments. The calibration proceeds in several steps, detailed below.

Throughout this subsection, we adopt the following notations:

- $\overline{X}$ denotes the empirical target associated with variable X;
- $[j_1; j_2]$ denotes the set $\{j \in \boldsymbol{J} \mid j_1 \leq j \leq j_2\}$ and $[j \leq j_2]$ denotes the set $\{j \in \boldsymbol{J} \mid j \leq j_2\}$;
- $\boldsymbol{\mu}_{[j_1;j_2]} \equiv \sum_{j \in \boldsymbol{J}} \mathbb{1}[j \in [j_1; j_2]] \mu_j$ will denote the population share of age group $[j_1; j_2]$;
- $\hat{H}_{[j_1;j_2]} \equiv \int_{\boldsymbol{S}} \mathbb{1}[j \in [j_1; j_2]] P(o \in \{E\} \mid s) \bar{h}_j \hat{\lambda}(ds)$ will denote the aggregate hours worked of age group $[j_1; j_2]$.
- $E\hat{m}p_{t,[j_1;j_2],\tilde{\omega}} \equiv \int_{\boldsymbol{S}} \mathbb{1}[j \in [j_1; j_2]; \omega = \tilde{\omega}] P_t(o \in \{E\} \mid s) \hat{\lambda}_t(ds)$ will denote the aggregate labour force of age group $[j_1; j_2]$ and, when specified, education group $\tilde{\omega}$ in period t.

C.2.1 Model initialization

We begin by setting all parameters whose calibration does not require solving the full model.

- **Age-specific productivity.** Age-specific labour productivity e_j follows a quadratic life-cycle profile, $e_j = a + b(j-18) + c(j-18)^2$. The coefficients (a, b, c) are chosen to satisfy three conditions. First, average productivity among young households (below age 30) is normalised to 1:

$$\frac{\sum_{j \in \boldsymbol{J}} \mathbb{1}[j \leq 30] e_j \mu_j}{\boldsymbol{\mu}_{[j \leq 30]}} = 1$$

Second, average productivity among middle-age households (30-49 years old) equals $\overline{e_{[j\in[30;49]]}}$:

$$\frac{\sum_{j\in \boldsymbol{J}} \mathbb{1}[j \in [30;49]] e_j \mu_j}{\mu_{[30;49]}} = \overline{e_{[j\in[30;49]]}}$$

Third, average productivity among older households (aged 50 and above) matches a given target $\overline{e_j[j \geq 50]}$:

$$\frac{\sum_{j\in \boldsymbol{J}} \mathbb{1}[j \geq 50] e_j \mu_j}{\mu_{[j\geq 50]}} = \overline{e_{[j\geq 50]}}$$

- **Capital depreciation and taxation.** Given targets for the capital–output ratio, $\overline{K_Y} \equiv \overline{\frac{\hat{K}}{(1+g)\hat{Y}}}$[20], and the pretax rental rate of capital, $\overline{r_k^{\text{pre}}}$, we recover the depreciation rate from the production function (A.71):

  $$\delta = \frac{\theta}{\overline{K_Y}} - \overline{r_k^{\text{pre}}}$$

  By definition, the post-tax rental rate r_k and the capital income tax rate τ^K satisfy

  $$(1+\tau^K) r_k = \overline{r_k^{\text{pre}}}, \quad \tau^K = \frac{\overline{T_Y^K}}{r_k \overline{K_Y}}$$

  Solving this system yields

  $$\tau^K = \frac{\overline{T_Y^K}}{\overline{r_k^{\text{pre}}}\,\overline{K_Y} - \overline{T_Y^K}}, \quad r_k = \overline{r_k^{\text{pre}}} - \frac{\overline{T_Y^K}}{\overline{K_Y}}$$

  Note that r_k is an equilibrium object rather than a structural parameter to be calibrated; accordingly, we include it as a calibration target in the next step described below.

- **Convenience yield.** Given the rate of return on capital claims, r_k, and a target for the annual real interest rate on public debt, $\overline{r_b}$, the no-arbitrage condition (A.73) pins down the convenience yield parameter as $\alpha = \frac{r_k - \overline{r_b}}{1+r_k}$.

C.2.2 Inner loop

Conditional on the parameters already calibrated and on a provisional value for the curvature of bequest utility, φ, all inner-loop parameters are jointly determined to match a vector of empirical targets. Specifically, 22 parameters are simultaneously calibrated to exactly hit 22 moments. Although, for ease of presentation, we assign each target to a corresponding parameter, the calibration is performed jointly and all parameters influence all moment conditions.

[20] In the main text, the targets are formulated for end-of-period capital and public debt, $\overline{K_t/Y_t}$ and $\overline{B_t/Y_t}$, which correspond to how these ratios are measured in the National Accounts. Here, instead, we express targets in terms of beginning-of-period variables, as is standard in the macroeconomic literature. Specifically, we define $\overline{K_Y} \equiv \overline{K_{t-1}/Y_t} = \frac{\overline{K_t/Y_t}}{1+g}$, $\overline{B_Y} \equiv \overline{B_{t-1}/Y_t} = \frac{\overline{B_t/Y_t}}{1+g}$.

- The **TFP** shifter Z is set to normalise aggregate output to one, $\overline{Y} \equiv 1$,

$$\hat{Y} = 1$$

- The **discount factor** β is chosen so that the rental rate r_k as calculated above is consistent with asset-market clearing,

$$\frac{\hat{S}}{\hat{Y}} = \overline{K_Y} + \overline{B_Y}$$

- The **bequest scaling parameter** ν_a is calibrated to match the bequests-to-GDP ratio, $\frac{\hat{\mathfrak{B}}}{\hat{Y}} = \overline{\mathfrak{B}_Y}$. Using equations (A.90), (A.86) and (A.92), this is equivalent to targeting beginning-of-period wealth relative to GDP,

$$\frac{\hat{A}}{(1+g)\hat{Y}} = \frac{\hat{S}}{(1+g)\hat{Y}} - \overline{\mathfrak{B}_Y}$$

- **Consumption and labour income tax rates** are set to match tax-revenue-to-GDP targets, $\overline{T_Y^C}$ and $\overline{T_Y^N}$:

$$\frac{\tau^c \hat{C}}{\hat{Y}} = \overline{T_Y^C}, \quad \frac{\hat{\mathcal{T}^{\mathcal{N}}}}{\hat{Y}} = \overline{T_Y^N}$$

- The **lump-sum transfer** $\hat{T}$ is chosen to replicate the public-consumption-to-GDP ratio, $\overline{G_Y}$:

$$\frac{\hat{G}}{\hat{Y}} = \overline{G_Y}$$

- **Social security**. The pension replacement-rate parameter ϕ^R is set to match pension expenditures relative to GDP, $\overline{T_Y^P}$:

$$\frac{\int_S \mathbb{1}[j \geq J_R] P(o \in \{I\} \mid s)\, \hat{w}\phi(\omega, j, o) \hat{\lambda}(ds)}{\hat{Y}} = \overline{T_Y^P}$$

. The transfer parameter for inactive non-retired households, ϕ^I, is chosen so that average disposable non-financial income for inactive households below retirement age represents a targeted fraction of average disposable non-financial income in the population, $\overline{T_Y^{\text{gmi}}}$:

$$\frac{\int_S \mathbb{1}[j < J_R] P(o \in \{I\} \mid s) \left(\hat{w}\phi(\omega, j, o) + \hat{T}\right) \hat{\lambda}(ds)}{\hat{Y}^{d,nf}} = \overline{T_Y^{\text{gmi}}}$$

where average disposable non-financial income is the sum of transfers received and labour income net of taxes: $\hat{Y}^{d,nf} = \hat{Tr} + \hat{w}\hat{N} - \hat{\mathcal{T}^{\mathcal{N}}}$.

- **Hours worked.** The inverse of hours worked per worker follows a quadratic life-cycle profile, $\bar{h}_j^{-1} = a + b(j-18) + c(j-18)^2$. The coefficients (a, b, c) are chosen to satisfy three conditions.

First, average hours worked per worker equal one,

$$\frac{\hat{H}}{\hat{Emp}} = 1$$

Second, average hours among young workers (ages 20–24) match their target, $\overline{h_{[j\in[20;24]]}}$:

$$\frac{\hat{H}[20;24]}{\hat{Emp}[20;24]} = \overline{h_{[j\in[20;24]]}}$$

Third, average hours among older workers (aged 50 and above) equals a given target, $\overline{h_{[j\geq 50]}}$:

$$\frac{\hat{H}[j\geq 50]}{\hat{Emp}[j\geq 50]} = \overline{h_{[j\geq 50]}}$$

- **labour force participation costs.** For each education group $\tilde{\omega}$, participation costs follow a quadratic life-cycle profile, $\kappa_{\tilde{\omega},j} = a_{\tilde{\omega}} + b_{\tilde{\omega}}(j-18) + c_{\tilde{\omega}}(j-18)^2$. The coefficients $(a_{\tilde{\omega}}, b_{\tilde{\omega}}, c_{\tilde{\omega}})$ are chosen so that labour force participation rates for ages 20–74, 20–24, and 25–59 match their respective targets for each education group, $\overline{LF_{[j\in[j_1;j_2],\tilde{\omega}]}}$:

$$\frac{\hat{Emp}_{[[20;74],\tilde{\omega}]}}{\pi^{\Omega}(\tilde{\omega})\boldsymbol{\mu}_{[20;74]}} = \overline{LF_{[j\in[20;74],\tilde{\omega}]}}, \quad \frac{\hat{Emp}_{[[20;24],\tilde{\omega}]}}{\pi^{\Omega}(\tilde{\omega})\boldsymbol{\mu}_{[20;24]}} = \overline{LF_{[j\in[20;24],\tilde{\omega}]}}, \quad \frac{\hat{Emp}_{[[25;59],\tilde{\omega}]}}{\pi^{\Omega}(\tilde{\omega})\boldsymbol{\mu}_{[25;59]}} = \overline{LF_{[j\in[25;59],\tilde{\omega}]}}$$

- **Participation elasticity.** The variance of the preference shock to work, σ^2_{χ}, is set to match a targeted aggregate labour participation elasticity among prime-age individuals (ages 25–61), denoted $\overline{lpe}$. This elasticity is defined as the impact response of aggregate participation to a fully transitory 1% increase in pre-tax wages, holding transfers fixed. Formally, starting from steady state, the economy is hit in period $t = 0$ by a fully transitory reduction in the labour tax level parameter equivalent to a 1% increase in wages[21]. Household policies are computed in partial equilibrium, assuming convergence back to steady state policies after at most $T = 150$ years. The elasticity is then given by

$$lpe \equiv 100\left(\frac{\hat{Emp}t,[25;61]}{\hat{Emp}[25;61]} - 1\right) = \overline{lpe}$$

Conditional on the calibration targets being satisfied, our strategy allows us to pin down the steady-state endogenous variables directly. In particular, r_k is obtained as described above, $\hat{\mathfrak{B}}$ is fixed to its target value, $\hat{\mathfrak{B}} = \overline{\mathfrak{B}_Y}$ (with the normalization $\overline{Y} = 1$) and $\Upsilon = \frac{\alpha \overline{B_Y}}{\overline{B_Y}+\overline{K_Y}}$ (which follows from the definition of Υ and (A.86)). Importantly, these objects are equilibrium outcomes rather than structural parameters, but they can be treated as known once the targets are met.

[21]A shock $d\tau^N = -\frac{(1-\xi)(1-\tau^N)}{100}$ has the same effect on after-tax wages as a 1% increase in pre-tax wages, while leaving transfers unchanged, as these are indexed to pre-tax wages.

As a consequence, for any given vector of inner-loop parameters, we do not need to solve for the implied steady state—i.e., we do not need to jointly determine $(r_k, \mathfrak{B}, \Upsilon)$. Instead, we compute policy functions, stationary distributions, and aggregate quantities conditional on the target values for $(r_k, \mathfrak{B}, \Upsilon)$, as described in Subsection C.1. We then evaluate calibration errors defined as absolute deviations between model-implied moments and their targets, $E_X = X - \overline{X}$[22]. We use a numerical solver to iterate on the inner-loop parameters to jointly minimise these errors until all targeted moments are matched with sufficient precision.

C.2.3 Outer loop

Finally, we calibrate the curvature of the bequest utility function (φ) using an outer-loop procedure. We begin by postulating a grid of candidate values for φ. For each candidate value, we recalibrate all inner-loop parameters as described above and solve for the corresponding steady state. Given each steady state conditional on φ, we compute the implied distribution of bequests among donors. We then select the value of φ that that minimises the quadratic distance between the model-implied and empirical Lorenz curves of bequests by donors.

D Transition

D.1 Problem Statement

The simulation exercises presented in this paper rely on the computation of a transition path between two steady states of the model described in the previous Sections. We begin by calibrating the trajectory of public expenditures financed by gradual a two-year increase in the statutory retirement age. This amounts to determining the path of $\frac{G_t}{Y_t}$, for a fixed debt-to-GDP ratio, that satisfies the government budget constraint as macroeconomic variables adjust endogenously.

Given this expenditure path, which is common across all scenarios considered, we then allow other fiscal instruments (one at a time) to clear the government budget constraint while keeping an unchanged debt-to-GDP ratio. As a result, the procedure used to solve for the transition path remains identical across scenarios; only the fiscal variable through which the adjustment operates differs from one scenario to another.

D.2 Solution Method

Once implemented, the different fiscal scenarios that we consider in this paper triggers a transition between the initial steady state and the new steady state. We solve for the transition through the following steps.

[22]The error term associated with β ensures financial market clearing; the error term associated with ν_a ensures consistency between bequests left and bequests received when financial markets clear; by construction, Υ is consistent with households' portfolios when financial markets clear.

We assume that the transition takes a finite number of periods T. To be clear, in period $t = t_0 - 1$ the economy is in the initial steady state; at $t = t_0$, the policy reform is enacted; at $t = t_0 + \mathsf{T}$, the economy is supposed to have reached its final steady state. Our task is then to find approximate paths for the endogenous variables for $t \in \{t_0, t_0 + 1, \ldots, t_0 + \mathsf{T} - 1\}$.

We postulate paths $\{r_{k,t}\}_{t=t_0}^{t_0+\mathsf{T}-1}$, $\{\hat{\mathfrak{B}}_t\}_{t=t_0}^{t_0+\mathsf{T}-1}$, $\{\Upsilon_t\}_{t=t_0}^{t_0+\mathsf{T}-1}$, and the relevant fiscal instrument $\{\Phi_t\}_{t=t_0}^{t_0+\mathsf{T}-1}$. From those variables, we can deduce the sequence of wages, asset returns and individual bequests.

Given $V_{t+1}^0(\cdot)$ and $V_{a,t+1}^0(\cdot)$ and $(r_{k,t}, \hat{\mathfrak{B}}_t, \Upsilon_t, \Phi_t)$, we have all the required ingredients to compute the decision rules $(g_{a,t}^j, g_{c,t}^j)$ and probability $P_t(o \mid s)$.

Thus starting from $V_{t0+\mathsf{T}}^0(\cdot)$ and $V_{a,t0+\mathsf{T}}^0(\cdot)$ and the postulated sequences $\{r_{k,t}\}_{t=t_0}^{t_0+\mathsf{T}-1}$, $\{\hat{\mathfrak{B}}_t\}_{t=t_0}^{t_0+\mathsf{T}-1}$, $\{\Upsilon_t\}_{t=t_0}^{t_0+\mathsf{T}-1}$, and $\{\Phi_t\}_{t=t_0}^{t_0+\mathsf{T}-1}$, one can compute sequences of decision rules $\{g_{a,t}^j, g_{c,t}^j\}_{t=t_0}^{t_0+\mathsf{T}-1}$ and probability $\{P_t(o \mid s)\}_{t=t_0}^{t_0+\mathsf{T}-1}$.

Next, given the sequence of decision rules on assets $\{g_{a,t}^j\}_{t=t_0}^{t_0+\mathsf{T}-1}$ and probability $\{P_t(o \mid s)\}_{t=t_0}^{t_0+\mathsf{T}-1}$, one can compute a sequence of stochastic kernels $\{\mathscr{Q}_{j,t}\}_{t=t_0}^{t_0+\mathsf{T}-1}$, from which, starting from the distribution at the beginning of period t_0 f_{j,t_0-1} (i.e., the initial steady-state distribution, before productivity shocks are realised and participation choices are made in t_0), one can iterate on (B.30) and (B.32) to obtain a sequence of distributions $\{f_{j,t}\}_{t=t_0}^{t_0+\mathsf{T}-1}$.

Using all the above ingredients, we can compute all aggregates and therefore the sequence of residuals for all $t \in \{t_0, t_{0+1}, \ldots, t_{0+\mathsf{T}-1}\}$, defined according to the formulas given in equations (B.41) to (B.44).

All in all, we have constructed a mapping

$$F : (\{r_{k,t}, \hat{\mathfrak{B}}_t, \Upsilon_t, \Phi_t\}_{t=t_0}^{t_0+\mathsf{T}-1}) \mapsto (\{E_{asset}, E_{bequest}, E_{portfolio}, E_{govbc}\}_{t=t_0}^{t_0+\mathsf{T}-1}).$$

We use an OLG-adjusted version of Auclert et al. (2021)'s Fake News algorithm to compute the Jacobian of F in the neighborhood of the final steady state, $\mathscr{J}_F$. Letting

$$\vec{X} \equiv \{r_{k,t}, \hat{\mathfrak{B}}_t, \Upsilon_t, \Phi_t\}_{t=t_0}^{t_0+T^f-1},$$

we then iterate on the quasi-Newton scheme

$$\vec{X}^{(k+1)} = \vec{X}^{(k)} - \mathscr{J}_F^{-1} F(\vec{X}^{(k)}).$$

We stop the process whenever the error terms $F(\vec{X}^{(k)})$ are all sufficiently close to zero.

E Open-economy model

In this Appendix, we spell out the open-economy variant of our model, in which equilibrium interest rates are determined by a residual asset demand curve emanating from the rest of the world.

E.1 Net foreign assets and asset market clearing

Let NFA_{t-1} denote the country's net foreign asset position at the beginning of time t. The asset market clearing condition (15) is then replaced with:

$$S_t - NFA_t = K_t + B_t \tag{E.1}$$

Here, $S_t - NFA_t = A_t + \Xi_t - NFA_t$ represents the demand for domestic assets and $K_t + B_t$ represents the supply of domestic assets.

We accommodate imperfect capital mobility by assuming that the domestic return to capital responds to the country's net foreign asset position according to the following relationship:

$$r_{k,t} = r_w - \Psi \times \frac{NFA_{t-1}}{Y_t}, \tag{E.2}$$

where r_w is the (exogenous, constant) return to capital in the rest of the world and $\Psi \in [0, \infty)$ measures the responsiveness of the domestic return to capital to changes in the domestic economy's foreign debt ($\Psi \to \infty$ corresponds to our baseline closed-economy model, where r_w accordingly becomes immaterial, while $\Psi \to 0$ is the small, fully-open economy limit wherein the domestic capital return must match the ROW's at every point in time).

In the open economy the liquidity variable Υ_t in equation (13) becomes:

$$\Upsilon_t = \frac{\alpha B_t}{S_t - NFA_t} \tag{E.3}$$

All the other equations of the open-economy model are identical to the baseline closed-economy model.

E.2 Calibration

We assume that the initial steady state of the open-economy model replicates that of the baseline closed-economy model. The calibration is thus the same as in Table 1, together with $r_w = r_k$ and $NFA = 0$. The other key parameter of the open-economy model is the degree of capital mobility Ψ. Alternative studies based on European economies provides a range of values for Ψ, going from 0.01 to 0.144 (Christoffel et al., 2008; Ratto et al., 2009; Adolfson et al., 2011, 2014). We follow Adolfson et al. (2014)'s estimate and set $\Psi = 0.04$.

E.3 Results

Tables 5-6 and Figures 9 and 12 provides the open-economy analogues of Tables 3-4 and Figures 14 and 15. By and large, the differences with our baseline results are minimal.

Table 5: Macroeconomic effects. (% change from initial to final BGP) - Open Economy

	Y	K/Y	TFP	$Empl.$	C
1 instrument					
Labour income tax (level)	−2.5	−2.8	0.5	−1.8	−4.3
Labour income tax (progressivity)	−1.8	−3.6	−0.6	1.3	−3.5
Capital tax	−1.7	−4.5	−0.5	2.1	−2.7
Consumption tax	0.3	−0.6	−0.3	1.2	−1.9
Retirement age +2y	2.3	−0.4	0.1	2.9	0.0
Pensions	1.5	0.5	0.2	1.1	−1.0
2 instruments: retirement age +1y and					
Labour income tax (level)	−0.0	−1.5	0.3	0.7	−2.0
Labour income tax (progressivity)	0.4	−1.9	−0.2	2.2	−1.6
Capital tax	0.4	−2.4	−0.2	2.6	−1.3
Consumption tax	1.3	−0.5	−0.1	2.1	−0.9
Pensions	1.9	0.0	0.1	2.0	−0.5

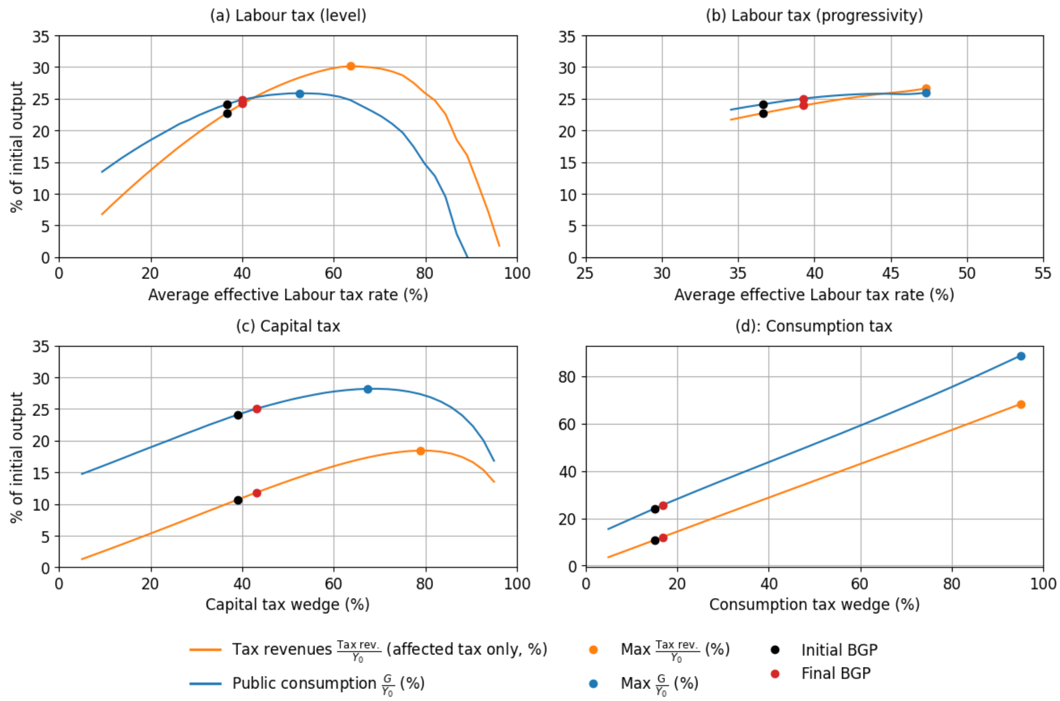
[(a)] See note of Table 3

Table 6: Changes in fiscal revenue. (pp. change from initial to final BGP) - Open Economy

	$\frac{\mathcal{T}^N}{Y}$	$\frac{\tau^K r^k K}{Y}$	$\frac{\tau^C C}{Y}$	$-\frac{\text{Tr}}{Y}$	$-\frac{r_b B}{Y}$
1 instrument					
Labour income tax (level)	**2.1**	0.1	−0.2	−0.5	−0.2
Labour income tax (progressivity)	**1.7**	0.1	−0.2	0.0	−0.3
Capital tax	−0.2	**1.3**	−0.1	0.3	0.0
Consumption tax	−0.0	0.0	**1.2**	0.2	−0.0
Retirement age +2y	−0.0	0.0	−0.2	**1.7**	−0.0
Pensions	0.0	−0.0	−0.3	**1.6**	0.0
2 instruments: retirement age +1y and					
Labour income tax (level)	**1.0**	0.1	−0.2	**0.7**	−0.1
Labour income tax (progressivity)	**0.8**	0.1	−0.2	**0.9**	−0.1
Capital tax	−0.1	**0.7**	−0.2	**1.0**	−0.0
Consumption tax	−0.0	0.0	**0.5**	**1.0**	−0.0
Pensions	−0.0	−0.0	−0.2	**1.6**	0.0

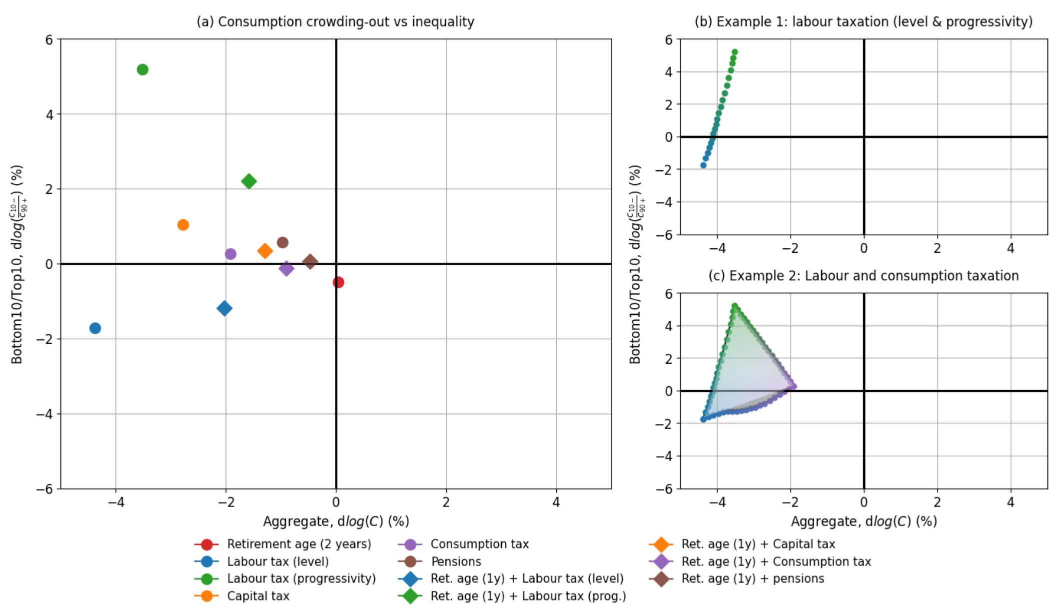
[(a)] See note of Table 4

Figure 14: Laffer curves - Open Economy



Notes: See notes of Figure 9

Figure 15: Aggregate crowding out vs changes in consumption inequality - Open Economy.



Notes: See notes of Figure 12